SECOND ORDER DIFFERENTIAL OPERATORS AND MODIFIED SZÁSZ-MIRAKJAN OPERATORS

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In this talk we present some generation results of strongly continuous positive semigroups for second order differential operators of the form

$$Lu(x) = xu''(x) + \beta(x)u'(x) + \gamma(x)u(x) \quad (x > 0) \tag{1}$$

in the framework of weighted continuous function spaces.
More precisely, we provide some conditions on the coefficients β and γ which guarantee generation property for the operator (1) equipped with Wentzell-type boundary conditions

$$\lim_{x\to 0^+} xu''(x) + \beta(x)u'(x) = 0 \text{ and } \lim_{x\to +\infty} \frac{xu''(x) + \beta(x)}{1 + x^m} = 0 \tag{2}$$

and with maximal-Wentzell type boundary conditions

$$\lim_{x\to 0^+} xu''(x) + \beta(x)u'(x) \in \mathbb{R} \text{ and } \lim_{x\to +\infty} \frac{xu''(x) + \beta(x)}{1 + x^m} = 0 \tag{3}$$

in the setting of the weighted spaces

$$E_m^* := \{u \in C(]0, +\infty[) : \lim_{\substack{x\to 0^+ \\ x\to +\infty}} \frac{u(x)}{1 + x^m} \in \mathbb{R}\}$$

and

$$E_m^0 := \{u \in C([0,+\infty[) : \lim_{x\to+\infty} \frac{u(x)}{1+x^m} = 0\}$$

$(m \geq 2)$.
Moreover, in the setting of E_m^0 we search a suitable sequence of positive linear operators whose iterates approximate the semigroup $(T(t))_{t\geq 0}$ above, and then the solutions to the initial-boundary value problem associated with the operator L

$$\begin{cases} \dfrac{\partial u}{\partial t} = Lu(x,t), \\ \\ u(x,0) = u_0(x), \\ u(\cdot,t) \text{ verifies (2) or (3)} \quad (\text{for every } t \geq 0). \end{cases} \tag{4}$$

To this purpose we construct a suitable sequence of operators by modifying the classical Szász-Mirakjan operators, as follows

$$S_n^*(f)(x) := \sum_{k=0}^{+\infty} e^{-nx} \frac{(nx)^k}{k!} \left(1 + \frac{\gamma(k/n)}{2n}\right) f\left(\frac{k}{n} + \frac{\beta(k/n)}{2n}\right)$$

and we show that $T(t) = \lim_{n\to\infty} (S_n^*)^{k(n)}$ strongly on E_m^0, for every $t \geq 0$ and for every sequence $(k(n))_{n\geq 1}$ of positive integers such that $\dfrac{k(n)}{n} \to t$. Such a representation formula allows us to derive some qualitative information on the semigroup and hence on the solutions to (4).

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