APPROXIMATION BY MAX-PRODUCT TYPE NONLINEAR OPERATORS

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The purpose of this talk is to present some approximation and shape preserving properties of the so-called nonlinear (more exactly sublinear) and positive, max-product operators, constructed by starting from any discrete linear approximation operators. The construction is based on a simple idea, exemplified for the case of Bernstein polynomials, as follows.

Let $B_n(f)(x) = \sum_{k=0}^{n} p_{n,k}(x) f(k/n)$ be with $p_{n,k}(x) = \binom{n}{k} x^k (1-x)^{n-k}$ and $f : [0,1] \to \mathbb{R}$. If in the obvious formula

$$B_n(f)(x) = \frac{\sum_{k=0}^{n} p_{n,k}(x) f(k/n)}{\sum_{k=0}^{n} p_{n,k}(x)}, x \in [0,1],$$

we replace the $\sum$ operator with the max operator denoted by $\bigvee$, then we obtain the so-called max-product Bernstein nonlinear (sublinear), piecewise rational operator

$$B_n^{(M)}(f)(x) = \frac{\bigvee_{k=0}^{n} p_{n,k}(x) f(k/n)}{\bigvee_{k=0}^{n} p_{n,k}(x)}, x \in [0,1],$$

where $\bigvee_{k=0}^{n} p_{n,k}(x)f(k/n) := \max_{0 \le k \le n}\{p_{n,k}(x)f(k/n)\}$.

The same idea of construction can be applied to any discrete linear Bernstein-type operator or to any discrete linear interpolation operator, obtaining thus the corresponding nonlinear max-product operators.

Surprisingly, the max-product operators do not lose the approximation properties of the corresponding linear operators to which they are attached. Moreover, for large classes of functions, they improve the order of approximation to the Jackson-type order. The most important improvement is in the case of interpolation (on any arbitrary system of nodes), when for the whole class of continuous functions the Jackson order $\omega_1(f; 1/n)$ is achieved. Also, the max-product Bernstein-type operators preserve the monotonicity and the quasi-convexity of the functions.

We will discuss here the main results for the max-product operators of : Bernstein-type, Favard-Szász-Mirakjan-type, truncated Favard-Szász-Mirakjan-type, Baskakov-type, truncated Baskakov-type, Meyer-König and Zeller-type, Bleimann-Butzer-Hahn-type, Hermite-Fejér interpolation-type on Chebyshev nodes of first kind, Lagrange interpolation-type on Chebyshev knots of second kind, Lagrange interpolation-type on arbitrary knots, generalized sampling-type, sampling sinc-type, Cardaliaguet-Euvrard neural network-type.

Open Problem. The saturation results for the max-product nonlinear operators are open questions.

The main results were obtained in a series of very recent papers jointly written with B. Bede and L. Coroianu.