HOMOGENIZATION IN DOMAINS WITH MIXED BOUNDARY CONDITIONS

Doina Cioranescu

Laboratoire J.-L. Lions, Université Pierre et Marie Curie, France
[cioran@ann.jussieu.fr]

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We study the asymptotic behavior as $\varepsilon \to 0$ of a class of second order elliptic problems in perforated domains with small holes distributed periodically with a period ε, and of size $r(\varepsilon)$ with $r(\varepsilon)/\varepsilon \to 0$. As an example, we consider the case where in each period, there are holes of size of order of $\varepsilon\delta_1(\varepsilon)$, and holes of size of order of $\varepsilon\delta_2(\varepsilon)$ with $\delta_1 \to 0$ and $\delta_2 \to 0$. We prescribe a homogeneous Dirichlet condition on the boundary of holes of size $\varepsilon\delta_2$ and a non homogeneous Neumann one on the boundary of holes of size $\varepsilon\delta_1$. The use of the periodic unfolding method allows us to consider general operators with highly oscillating (with ε) coefficients. Suppose that δ_1 and δ_2 are such that there exist k_1 and k_2 with

$$k_1 = \lim_{\varepsilon\to 0} \frac{\delta_1^{n-1}}{\varepsilon}, \quad 0 \le k_1 < \infty \quad \text{and } k_2 = \lim_{\varepsilon\to 0} \frac{\delta_2^{n/2-1}}{\varepsilon}, \quad 0 \le k_2 < \infty,$$

where $n \ge 3$ is the dimension of the space. The limit problem contains two additional terms: a zero order one (depending on k_2) representing the contributions of Dirichlet holes, and a second extra right-hand side term (depending on k_1) generated by the Neumann holes. Several other situations are discussed, in particular that mixing homogeneous and nonhomogeneous Neumann conditions.