ON QUATERNION VALUED GAUSSIAN RANDOM VARIABLES

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The main result of this work is the formulation and proof of Polya's theorem on the characterization of Gaussian random variables with values in quaternion algebra in which three types of Gaussian random variables are considered: real, complex and quaternion Gaussian random variables. The present work is closely related with the results of N. Vakhania where Polya's theorem is formulated for the case of complex random variables. It was shown that Polya's type condition characterizes complex Gaussian random variables.

Theorem. *Let ξ be a quaternion random variable, $\xi_1, \xi_2, ..., \xi_n$, $n \geq 2$ be pairwise independent random variables, that have the same distribution as ξ, and $\{a_1, a_2, ..., a_n\}$ be nonzero quaternions that form the jointly quaternion system and satisfy the condition $\sum_{h=1}^{n} |a_h|^2 = 1$. Then, if the sum $\eta = \sum_{h=1}^{n} a_h \xi_h$ has the same distribution as ξ, ξ is quaternion Gaussian random variable. Therefore, characteristic function of the random variable ξ has the form $\chi_\xi(q) = exp(-\frac{1}{8}|q|^2 E|\xi|^2)$.*