APPROXIMATION PROBLEMS BY POSITIVE LINEAR OPERATORS IN FUNCTION SPACES ON UNBOUNDED DOMAINS

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The talk will be devoted to discuss some new Korovkin-type theorems, established in [1] and [2], which hold true in the setting of weighted continuous function spaces $C_0^w(X)$ and $C_*^w(X)$ as well as in that one of $L^p(X, \mu)$ spaces, $1 \leq p < +\infty$, where X is a locally compact Hausdorff space, w a weight on X and μ a regular positive Borel measure on X. In particular, these results furnishe simple tools to easily construct Korovkin subsets in $C_0^w(\mathbb{R}^N)$ and $L^p(\mathbb{R}^N, \mu)$.

Among other things, we show that, if μ is a positive Borel measure on an unbouded locally compact subset of $\mathbb{R}^N$ and if $f_0 \in C(X) \cap L^p(X, \mu)$ is a strictly positive function such that $\| \cdot \|^2 f_0 \in L^p(X, \mu)$ for some $1 \leq p < +\infty$, then the subset $\{f_0, f_0 pr_1, ..., f_0 pr_N, \| \cdot \|^2 f_0\}$ is a Korovkin subset in $L^p(X, \mu)$ (here pr_i denotes the i-th coordinate function on X, $1 \leq i \leq N$).

In particular, if μ is finite and $\|\cdot\|^2 \in L^p(X, \mu)$, then $\{\mathbf{1}, pr_1, ..., pr_N, \|\cdot\|^2\}$ is a Korovkin subset in $L^p(X, \mu)$.

We shall discuss some applications which mainly concern the approximation properties on $C_0^w(\mathbb{R}^N)$ and on $L^p(\mathbb{R}^N, \mu)$ of a sequence of positive linear operators which generalize Gauss-Weierstrass operators and which are defined by

$$G_n(f)(x) := \left(\frac{n}{4\pi\alpha(x)}\right)^{\frac{N}{2}} \int_{\mathbb{R}^N} f(t)e^{-\frac{n}{4\alpha(x)}\|t-x\|^2}dt,$$

for every real valued Borel measurable function f on $\mathbb{R}^N$ for which the integral to the right-hand side is absolutely convergent. Here $\alpha : \mathbb{R}^N \to \mathbb{R}$ is a strictly positive continuous function.

These operators have been introduced and studied in [2]. Among other things, we show that they are a useful tool to approximate not only continuous functions and L^p- functions, but also the positive semigroups (and hence the solutions of the degenerate diffusion equations) related to multiplicative perturbations of the Laplacian ([3]).

REFERENCES

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