Laboratory 4: Partial Differential Equations

1. Compute first order and second order partial derivatives for the following functions:

   (a) $f(x,y) = x^2 + 5xy + 6y^4 + 10y$;

   (b) $f(x,y) = e^{xy} - \sin(xy)$;

   (c) $f(x,y,z) = xyz - x^2y^2z^2$;

   (d) $f(x,y,z) = \cos(xyz) + \tan(x) - e^{\ln(y)}$.

2. Compute first order and second order partial derivatives and evaluate them in the specified points:

   (a) $f(x,y) = x^2 + 5xy + 6y^4 + 10y$ in $(3,2)$

   (b) $f(x,y) = e^{x^2y^2} - 5x^2y^2$ in $(1,1)$

   (c) $f(x,y) = \tan(xy) + e^{xy}$ in $(0,1)$

   (d) $f(x,y,z) = \cos(xyz) + \tan(x) - e^{\ln(y)}$ in $(1,0,1)$

3. Let's consider the function:

$$f(x,y) = \frac{x^2 + y^2}{xy}$$

   Find: $\frac{\partial^3 f}{\partial x^2 \partial y}(1,2)$, $\frac{\partial^3 f}{\partial x \partial y^2}(1,1)$, $\frac{\partial^4 f}{\partial x^2 \partial y^2}(1,2)$, $\frac{\partial^4 f}{\partial x \partial y^3}(2,1)$.

4. Find the general solution for the following first order partial differential equations:

   (a) $(x+2y)\frac{\partial f}{\partial x} - y\frac{\partial f}{\partial y} = 0$;

   (b) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 0$;

   (c) $(x)\frac{\partial f}{\partial x} - 2y\frac{\partial f}{\partial y} - z\frac{\partial f}{\partial z} = 0$;

   (d) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + xy\frac{\partial f}{\partial z} = 0$;

   (e) $(y^2 + z^2 - x^2)\frac{\partial f}{\partial x} - 2xy\frac{\partial f}{\partial y} - 2xz\frac{\partial f}{\partial z} = 0$;

5. Find the solution for the following second order partial differential equations:

   (a) $u_{xx} - 4u_{yy} = 0$;
   (b) $4u_{xx} - 4u_{xy} - 2u_{yy} = 0$;
   (c) $x^2 u_{xx} + 4y^2 u_{yy} = 0$
   (d) $u_{xx} + x^2 y^2 u_{yy} = 0$;

6. Let's consider the following partial differential equation:

   $xf_y - yf_x = 0$

   (a) Find the general solution.
   (b) Plot the graph of the particular solution corresponding to the following generator functions: $F(t) = ln(t)$, $F(t) = e^t$, $F(t) = \sin(t)$.