Babeş-Bolyai University, Cluj-Napoca
Faculty of Mathematics and Computer Science

Research report/2008

UEFISCSU-CNCSIS PN-ID-525/2007: THE STUDY OF SOME VISCOUS FLUID FLOWS IN POROUS MEDIA WITH APPLICATIONS IN BIOLOGY AND MEDICINE

Objectives/2008

1. The study of viscous incompressible fluids in isotropic porous media or in the presence of isotropic porous particles, in domains with Lyapunov boundaries, by using boundary integral and singularity methods. The analysis of the associated boundary value problems in Holder spaces of functions.

2. Mathematical models and theoretical analysis of certain steday/unsteady incompressible viscous fluid flows and heat transfer problems on surfaces with a given geometry embedded in viscous fluids, micropolar and porous media, by using the approximations of boundary layer theory

3. Numerical analysis of steday/unsteady incompressible viscous fluid flows and heat transfer problems on surfaces with a given geometry embedded in viscous fluids, micropolar and porous media, by using the approximations of boundary layer theory

Research papers

1. **M. Kohr**, G.P. Raja Sekhar, W.L. Wendland, *Boundary integral equations for a three dimensional Stokes-Brinkman cell model*, Mathematical Models and Methods in Applied Sciences, vol. **19**(2009), to appear (ISI)

2. **M. Kohr**, W.L. Wendland, G.P. Raja Sekhar, *Boundary integral equations for two-dimensional low Reynolds number flow past a porous body*, Mathematical Methods in the Applied Sciences, Published online: Sep 26 2008. DOI: 10.1002/mma.1074 (ISI).

3. **M. Kohr**, **G. Kohr**, W.L. Wendland, *Boundary integral equations for viscous incompressible flows in porous media or past porous bodies*, Proceedings in Applied Mathematics and Mechanics, vol. **8** (2008), to appear (BDI).

4. **C. Revnic**, **T. Groşan**, I. Pop, D.B. Ingham, *Free convection in a square cavity filled with a bidisperse porous medium*, submitted to International Journal of Thermal Sciences (ISI).

5. S.R. Pop, **T. Groşan**, I. Pop, *Effect of variable viscosity on free convection flow in a horizontal porous channel with a partly heated or cooled wall*, submitted to Revista de Chimie (ISI).

6. **D. S. Cîmpean**, N. Lungu, I. Pop, *A problem of entropy generation in a channel filled with a porous medium*, submitted.

7. **D. S. Cîmpean**, I. Pop, *Steady flow of a micropolar fluid in a sinusoidal channel*, submitted.

Abstracts of the papers

The purpose of the paper [1] is to prove the existence and uniqueness of the solution in Sobolev or Hölder spaces for a cell model problem which describes the Stokes flow of a viscous incompressible fluid in a bounded region past a porous particle. The flow within the porous particle is described by the Brinkman equation. In order to obtain the desired existence and uniqueness result, we use an indirect boundary integral formulation and potential theory for both Brinkman and Stokes equations. Some special cases, which refer to the cell model for a porous particle with large permeability, or to the exterior Stokes flow past a porous particle, are also presented.

In the paper [2] we use the method of matched asymptotic expansions in order to study the two-dimensional steady flow of a viscous incompressible fluid at low Reynolds number past a porous body of arbitrary shape. One assumes that the flow inside the porous body is described by the Brinkman model, i.e. by the continuity and Brinkman equations, and that the velocity and boundary traction fields are continuous across the interface between the fluid and porous media. By considering some indirect boundary integral representations, the inner problems are reduced to uniquely solvable systems of Fredholm integral equations of the second kind in some Sobolev or Holder spaces, while the outer problems are solved by using the singularity method. It is shown that the force exerted by the exterior flow on the porous body admits an asymptotic expansion with respect to low Reynolds number, whose terms depend on the solutions of the above mentioned system of boundary integral equations. In addition, the case of small permeability of the porous body is also treated.

The purpose of the paper [3] is to present the existence and uniqueness result for a boundary value problem which describes the flow of a viscous incompressible fluid past an arbitrary porous particle with a Lyapunov boundary, which is embedded in a second porous medium, by using the Brinkman model and the potential theory. We use a boundary integral method that reduces the problem to a system of second kind Fredholm integral equations that has a unique solution in some Hölder spaces.

In the paper [4] the classical problem of steady Darcy free convection in a square cavity filled with a porous medium has been extended to the case of a bidisperse porous medium (BDPM) by following the recent model proposed by Nield and Kuznetsov [D.A. Nield, A.V. Kuznetsov, Natural convection about a vertical plate embedded in a bidisperse porous medium, Int. J. Heat Mass Transfer 51(2008) 1658-1664] and Rees et al. [D.A.S. Rees, D.A. Nield, A.V. Kuznetsov, Vertical free convective boundary-layer flow in a bidisperse porous medium, ASME J. Heat Transfer 130(2008) 1-9]. The transformed partial differential equations in terms of the dimensionless stream function and temperature are solved numerically using a finite difference method for some values of the governing parameters when the Rayleigh number Ra is equal to 10^2 and 10^3. Results are presented for the flow field with streamlines, temperature field by isotherms and heat transfer by local and mean Nusselt numbers are presented for both the f- and p-phases. It is found that the most important parameters that influence the fluid flow and heat transfer are the inter-phase heat transfer parameter H and the modified thermal conductivity ratio parameter γ.

A theoretical study of the effect of variable viscosity on the steady free convection flow in a horizontal infinite porous channel when a part of its bottom wall is heated is presented in the paper [5]. The transformed equations are solved numerically using a finite-difference method. The effects of the Rayleigh number and viscosity parameter on the flow and heat transfer characteristics are discussed

The problem studied in the paper [6] is that of entropy generation for mixed convection in an inclined channel. The channel is filled with a porous medium and has an uniform wall heat flux. The flow is upward and the heat flux is into the channel. The solutions of the governing Darcy and energy equations are used for analyzing the entropy generation and the Bejan number into the channel. The results are plotted and studied for different important parameters involved and for different inclinations angle of the channel.

A problem of a steady micropolar flow in a two-dimensional sinusoidal channel is studied in the paper [7]. The governing equations of the flow and the boundary conditions involved in the problem are presented. The stream function of the flow and the gyration vector are expanded in a series thereby the wall amplitude is the perturbation parameter. In order to simplify the boundary conditions of the problem on the wall, the walls of the channel are transformed into parallel straight lines. For the case of a weak concentration of particles, considered, the problem is reduced to a system of ordinary differential equations, which is solved numerically using the NAG Fortran routine $D02HBF$. Representative results for the velocity profiles into the channel are plotted and presented for different Reynolds number and different material parameter. A very good agreement can be observed by comparing these results with previous results obtained by Tsangaris and Leiter (1984), for the case of a clear Newtonian fluid. Also, the microrotation behavior and the stream function are studied for various sections of the channel and the shear stress is calculated in its non-dimensional form and observed.

Most of the above results were presented to national/international conferences, and as seminar talks to prestigious universities abroad (University of Toronto, Stuttgart University, Free University of Berlin), that were visited by some of the members of the present grant.

Director
Professor Mirela Kohr