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SOME (Λ, b)-TYPE MAPPINGS IN TOPOLOGICAL SPACES

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Abstract. In this paper, the authors introduce and study (Λ, b)-continuous, (Λ, b)-irresolute and quasi-(Λ, b)-irresolute mappings. Some characterizations and several properties concerning aforesaid mappings are obtained. The authors also introduce (Λ, b)-compactness and (Λ, b)-connectedness. It is proved that (Λ, b)-compactness (resp. (Λ, b)-connectedness) is preserved under (Λ, b)-irresolute mappings. The paper also touches the topics frontier points, Dirichlet's function, filter and algebraic structure of some functions.

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Key words. Λ_b-set, (Λ, b)-closed set, (Λ, b)-open set, b-continuous function, b-irresolute function.

1. INTRODUCTION

Maki [12] introduced the notion of Λ-sets and Andrijević [1] introduced the b-open sets in topological spaces. In [4], Caldas et al. defined and investigated Λ_b-sets using b-open sets. Via Λ_b-sets and b-closed sets, Boonpok [2] introduced (Λ, b)-closed sets and investigated several properties in topological spaces. In this paper, we introduce concepts of (Λ, b)-continuous, (Λ, b)-irresolute, quasi-(Λ, b)-irresolute mappings and study several behaviours and characterizations. We also introduce (Λ, b)-compactness and (Λ, b)-connectedness and relate them with (Λ, b)-continuous, (Λ, b)-irresolute mappings. We show that (Λ, b)-irresolute image of (Λ, b)-compact (resp. (Λ, b)-connected) space is (Λ, b)-compact (resp. (Λ, b)-connected).

2. PRELIMINARIES

Throughout this paper, by (X, τ), (Y, σ) and (Z, η) (or simply X, Y and Z) we mean topological spaces in which, unless explicitly mentioned, any kind of separation axioms are not considered. From now, by space we understood topological space. For $A \subseteq X$, $\mathrm{Int}(A)$, $\mathrm{Cl}(A)$ and $X \setminus A$ are used to denote interior, closure and complement of A respectively. For $x \in X$, $\tau(x)$ stands for the collection of all open sets containing x.

A subset A of a space X is called b-open [1] or γ-open [9] if $A \subseteq \mathrm{Cl}(\mathrm{Int}(A)) \cup \mathrm{Int}(\mathrm{Cl}(A))$. Complement of a b-open set is called b-closed. The b-closure (resp.

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b-interior) of A, denoted by $b\mathrm{Cl}(A)$ [1] or $\mathrm{Cl}_b(A)$ [3] (resp. $b\mathrm{Int}(A)$ [1] or $\mathrm{Int}_b(A)$ [3]), is the smallest (resp. largest) b-closed (resp. b-open) set containing (resp. contained in) A. The family of all b-open (resp. b-closed) sets in X is denoted as $BO(X,\tau)$ (resp. $BC(X,\tau)$). In [4], the subset A^{Λ_b} (resp. A^{V_b}) is defined as the intersection (resp. union) of all b-open (resp. b-closed) subsets of X containing (resp. contained in) A. It is noticeable that A^{Λ_b} is denoted as $b\mathrm{Ker}(A)$ in [3] and γ-$\mathrm{Ker}(A)$ in [6]. A is called a Λ_b-set (resp. V_b-set) [4] if $A^{\Lambda_b} = A$ (resp. $A^{V_b} = A$). Furthermore, the authors Caldas *et al.* in [4] have shown that for subsets A and B of a space X, (i) $A \subseteq B$ implies $A^{V_b} \subseteq B^{V_b}$; (ii) $(X \setminus A)^{\Lambda_b} = X \setminus A^{V_b}$; (ii) for $A \in BO(X,\tau)$; A is a Λ_b-set and (iv) A is a Λ_b-set if and only if $X \setminus A$ is a V_b-set.

In this paragraph we discuss some notations and terminologies of [2]. A subset A of a space X is called (Λ,b)-closed if $A = T \cap C$, where T is a Λ_b-set and C is b-closed set. Complement of a (Λ,b)-closed set is called (Λ,b)-open. The family of (Λ,b)-closed (resp. (Λ,b)-open) subsets of X is denoted as $\Lambda_b C(X,\tau)$ (resp. $\Lambda_b O(X,\tau)$). The (Λ,b)-closure (resp. (Λ,b)-interior) of A, denoted by $A^{(\Lambda,b)}$ (resp. $A_{(\Lambda,b)}$) is defined in analogous manner of $\mathrm{Cl}(A)$ (resp. $\mathrm{Int}(A)$). The symbol $\Lambda_b C(X,x)$ (resp. $\Lambda_b O(X,x)$) denotes the family of all (Λ,b)-closed (resp. (Λ,b)-open) sets containing x. The subset $\Lambda_{(\Lambda,b)}(A)$ is defined as $\Lambda_{(\Lambda,b)}(A) = \bigcap\{U \in \Lambda_b O(X,\tau) : A \subseteq U\}$. Again, we learnt from [2] that every Λ_b-set (resp. b-closed set) is (Λ,b)-closed; and for subsets A and B of a space X, (i) $A \subseteq B$ implies $A^{(\Lambda,b)} \subseteq B^{(\Lambda,b)}$; (ii) $[A^{(\Lambda,b)}]^{(\Lambda,b)} = A^{(\Lambda,b)}$; (iii) A is (Λ,b)-closed if and only if $A = A^{(\Lambda,b)}$; (iv) A is (Λ,b)-open if and only if $A = A_{(\Lambda,b)}$; and (v) $A^{(\Lambda,b)}$ (resp. $A_{(\Lambda,b)}$) is (Λ,b)-closed (resp. (Λ,b)-open).

PROPOSITION 2.1. *The following statements are valid for a space X:*

(1) *Every b-open set is (Λ,b)-open.*
(2) *Every b-closed set is (Λ,b)-open.*
(3) *Every V_b-set is (Λ,b)-open.*

THEOREM 2.2. *For a subset A of a space X, the following are equivalent:*

(1) *A is (Λ,b)-open;*
(2) *$A = P \cup Q$, where P is a V_b-set and Q is a b-open set;*
(3) *$A = P \cup \mathrm{Int}_b(A)$;*
(4) *$A = A^{V_b} \cup \mathrm{Int}_b(A)$;*
(5) *$A = A^{V_b} \cup A_{(\Lambda,b)}$.*

PROPOSITION 2.3. *Let A be a subset of a space X and $x \in X$. Then $x \in \Lambda_{(\Lambda,b)}(A)$ if and only if $A \cap F \neq \emptyset$ for every $F \in \Lambda_b C(X,x)$.*

COROLLARY 2.4. *For a subset A of a space X, $\Lambda_{(\Lambda,b)}(A) = \{x \in X : \{x\}^{(\Lambda,b)} \cap A \neq \emptyset\}$.*

PROPOSITION 2.5. *Let X be a space and $x \in X$. Then $y \in \Lambda_{(\Lambda,b)}(\{x\})$ if and only if $x \in \{y\}^{(\Lambda,b)}$.*

Proposition 2.6. *Let X be a space. Then for every $x \in X$, $\Lambda_{(\Lambda,b)}(\{x\}) \neq X$ if and only if $\bigcap\{\{x\}^{(\Lambda,b)} : x \in X\} = \emptyset$.*

We close our this short section with the following theorem:

Theorem 2.7. *For any two points x and y of a space X, the following are equivalent:*

(1) $\Lambda_{(\Lambda,b)}(\{x\}) \neq \Lambda_{(\Lambda,b)}(\{y\})$;
(2) $\{x\}^{(\Lambda,b)} \neq \{y\}^{(\Lambda,b)}$.

Proof. Let $\Lambda_{(\Lambda,b)}(\{x\}) \neq \Lambda_{(\Lambda,b)}(\{y\})$. Then we can find $p \in X$ such that $p \in \Lambda_{(\Lambda,b)}(\{x\})$ but $p \notin \Lambda_{(\Lambda,b)}(\{y\})$. Using Proposition 2.5 from $p \in \Lambda_{(\Lambda,b)}(\{x\})$, we get $x \in \{p\}^{(\Lambda,b)}$ and hence $\{x\}^{(\Lambda,b)} \subseteq \{p\}^{(\Lambda,b)}$. Again using Proposition 2.5 from $p \notin \Lambda_{(\Lambda,b)}(\{y\})$, we get $y \notin \{p\}^{(\Lambda,b)}$ and hence $y \notin \{x\}^{(\Lambda,b)}$. Hence $\{x\}^{(\Lambda,b)} \neq \{y\}^{(\Lambda,b)}$. Conversely, let $\{x\}^{(\Lambda,b)} \neq \{y\}^{(\Lambda,b)}$. Then we can find $t \in X$ such that $t \in \{x\}^{(\Lambda,b)}$ but $t \notin \{y\}^{(\Lambda,b)}$. From $t \in \{x\}^{(\Lambda,b)}$ and Proposition 2.5, we have $x \in \Lambda_{(\Lambda,b)}(\{t\})$. Therefore $\{x\} \subseteq \Lambda_{(\Lambda,b)}(\{t\})$ implies $\Lambda_{(\Lambda,b)}(\{x\}) \subseteq \Lambda_{(\Lambda,b)}[\Lambda_{(\Lambda,b)}(\{t\})] = \Lambda_{(\Lambda,b)}(\{t\})$, by Lemma 3.36 of [2]. Now using Proposition 2.5 from $t \notin \{y\}^{(\Lambda,b)}$, we have $y \notin \Lambda_{(\Lambda,b)}(\{t\})$. Clearly $y \notin \Lambda_{(\Lambda,b)}(\{x\})$. Hence $\Lambda_{(\Lambda,b)}(\{x\}) \neq \Lambda_{(\Lambda,b)}(\{y\})$. □

3. (Λ, b)-CONTINUOUS, (Λ, b)-IRRESOLUTE AND QUASI-(Λ, b)-IRRESOLUTE FUNCTIONS

In this section we introduce (Λ, b)-continuous, (Λ, b)-irresolute and quasi-(Λ, b)-irresolute mappings and study some properties and characterizations.

Definition 3.1. Let X and Y be two spaces. A function $f : X \to Y$ is said to be

(1) (Λ, b)-continuous (resp. b-continuous or γ-continuous [9]) if for every open subset V of Y, $f^{-1}(V)$ is (Λ, b)-open (resp. b-open) in X.
(2) (Λ, b)-irresolute (resp. b-irresolute or γ-irresolute [5, 8]) if for every (Λ, b)-open (resp. b-open) subset V of Y, $f^{-1}(V)$ is (Λ, b)-open (resp. b-open) in X.
(3) quasi-(Λ, b)-irresolute if for every b-open subset V of Y, $f^{-1}(V)$ is (Λ, b)-open in X.

The following examples illustrate the existence of (Λ, b)-continuous, (Λ, b)-irresolute and quasi-(Λ, b)-irresolute functions.

Example 3.2. Consider the real line $\mathbb{R}$ endowed with the usual topology τ_u. The well known Dirichlet's function $f : (\mathbb{R}, \tau_u) \to (\mathbb{R}, \tau_u)$ defined by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases} \tag{1}$$

is (Λ, b)-continuous on $\mathbb{R}$.

Example 3.3. Let $X = Y = \mathbb{R}$, τ_u and τ_d be respectively the usual and discrete topology on $\mathbb{R}$. The function $f : (X, \tau_u) \to (Y, \tau_d)$ defined in (1) is (Λ, b)-irresolute as well as quasi-(Λ, b)-irresolute.

Theorem 3.4. *For a function $f : X \to Y$, the following are equivalent:*

(1) *f is (Λ, b)-continuous;*
(2) *for every closed subset F of Y, $f^{-1}(F) \in \Lambda_b C(X, \tau)$;*
(3) *for each $x \in X$ and for every $V \in \sigma(f(x))$, there is a $U \in \Lambda_b O(X, x)$ such that $f(U) \subseteq V$;*
(4) *for every $A \subseteq X$, $f[A^{(\Lambda,b)}] \subseteq \mathrm{Cl}(f(A))$;*
(5) *for every $B \subseteq Y$, $[f^{-1}(B)]^{(\Lambda,b)} \subseteq f^{-1}(\mathrm{Cl}(B))$;*
(6) *for every $B \subseteq Y$, $f^{-1}(\mathrm{Int}(B)) \subseteq [f^{-1}(B)]_{(\Lambda,b)}$.*

Proof. (1) $\Longleftrightarrow$ (2): Since $Y \setminus F$ is open and f is (Λ, b)-continuous, $X \setminus f^{-1}(F) = f^{-1}(Y \setminus F)$ is (Λ, b)-open, witnessing that $f^{-1}(F) \in \Lambda_b C(X, \tau)$. Conversely, let V be any open subset of Y. Then $Y \setminus V$ is closed in Y. By hypothesis, $X \setminus f^{-1}(V) = f^{-1}(Y \setminus V)$ is (Λ, b)-closed and hence $f^{-1}(V)$ is (Λ, b)-open in X. Hence f is (Λ, b)-continuous.

(1) $\Longleftrightarrow$ (3): Let V be an open subset of Y and $f(x) \in V$. Then $x \in f^{-1}(V)$. Consider $U = f^{-1}(V)$. Since f is (Λ, b)-continuous, U is a (Λ, b)-open subset of X such that $x \in U$ and $f(U) \subseteq V$. Conversely, let V be any open subset of Y and $x \in f^{-1}(V)$. Then $f(x) \in V$. By assumption, there exists a (Λ, b)-open subset U_x of X such that $x \in U_x$ and $f(U_x) \subseteq V$. Hence $f^{-1}(V) = \bigcup\{U_x : x \in f^{-1}(V)\}$. Therefore $f^{-1}(V)$ is (Λ, b)-open in X, by Theorem 3.5 of [2]. Hence f is (Λ, b)-continuous.

(2) $\Longleftrightarrow$ (4): Since $\mathrm{Cl}(f(A))$ is closed in Y, $f^{-1}(\mathrm{Cl}(f(A)))$ is (Λ, b)-closed in X, by (2). Now $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\mathrm{Cl}(f(A)))$ implies that $A^{(\Lambda,b)} \subseteq f^{-1}(\mathrm{Cl}(f(A)))$. Hence $f[A^{(\Lambda,b)}] \subseteq \mathrm{Cl}(f(A))$. Conversely, let F be a closed subset of Y. By hypothesis, $f([f^{-1}(F)]^{(\Lambda,b)}) \subseteq \mathrm{Cl}(f(f^{-1}(F))) \subseteq \mathrm{Cl}(F) = F$. Therefore $[f^{-1}(F)]^{(\Lambda,b)} \subseteq f^{-1}(F)$. Moreover, $f^{-1}(F) \subseteq [f^{-1}(F)]^{(\Lambda,b)}$. Thus $f^{-1}(F) = [f^{-1}(F)]^{(\Lambda,b)}$ and hence $f^{-1}(F)$ is (Λ, b)-closed in X.

(4) $\Longleftrightarrow$ (5): Let B be a subset of Y. By assumption, $f[(f^{-1}(B))^{(\Lambda,b)}] \subseteq \mathrm{Cl}(f(f^{-1}(B))) \subseteq \mathrm{Cl}(B)$. Hence $[f^{-1}(B)]^{(\Lambda,b)} \subseteq f^{-1}(\mathrm{Cl}(B))$. Conversely, let A be a subset of X. Then by assumption, $[f^{-1}(f(A))]^{(\Lambda,b)} \subseteq f^{-1}(\mathrm{Cl}(f(A)))$. Since $A \subseteq f^{-1}(f(A))$, $A^{(\Lambda,b)} \subseteq [f^{-1}(f(A))]^{(\Lambda,b)}$. Thus $A^{(\Lambda,b)} \subseteq f^{-1}(\mathrm{Cl}(f(A)))$ and hence $f[A^{(\Lambda,b)}] \subseteq \mathrm{Cl}(f(A))$.

(1) $\Longleftrightarrow$ (6): For any $B \subseteq Y$, $\mathrm{Int}(B)$ is open in Y and hence by (1), $f^{-1}(\mathrm{Int}(B))$ is (Λ, b)-open in X and is contained in $f^{-1}(B)$. So $f^{-1}(\mathrm{Int}(B)) \subseteq [f^{-1}(B)]_{(\Lambda,b)}$. Conversely, let V be open in Y. Then $V = \mathrm{Int}(V)$ implies $f^{-1}(V) = f^{-1}(\mathrm{Int}(V)) \subseteq [f^{-1}(V)]_{(\Lambda,b)}$, by (6). Also $[f^{-1}(V)]_{(\Lambda,b)} \subseteq f^{-1}(V)$. Thus $f^{-1}(V) = [f^{-1}(V)]_{(\Lambda,b)}$ and hence $f^{-1}(V)$ is (Λ, b)-open in X. Therefore f is (Λ, b)-continuous. $\square$

Recall that kernel of a subset A [13] of a space X is the set $\text{Ker}(A) = \bigcap\{U \in \tau : A \subseteq U\}$. In [12], $\text{Ker}(A)$ is denoted by A^{Λ}.

Lemma 3.5 ([10]). *Let A be a subset of a space X. Then $x \in \text{Ker}(A)$ if and only if $A \cap F \neq \emptyset$ for every closed set F containing x.*

Theorem 3.6. *Let $f : X \to Y$ be a (Λ, b)-continuous function. Then for every $A \subseteq X$, $f[\Lambda_{(\Lambda,b)}(A)] \subseteq \text{Ker}(f(A))$.*

Proof. Suppose $y \notin \text{Ker}(f(A))$. By Lemma 3.5, there exists a closed set F in Y such that $y \in F$ and $f(A) \cap F = \emptyset$. Now $A \cap f^{-1}(F) \subseteq f^{-1}(f(A)) \cap f^{-1}(F) = f^{-1}(f(A) \cap F) = \emptyset$ implies $A \cap f^{-1}(F) = \emptyset$. Since f is (Λ, b)-continuous function, $f^{-1}(F)$ is (Λ, b)-closed in X. Moreover, $f^{-1}(y) \subseteq f^{-1}(F)$. Therefore, by Proposition 2.3, $x \notin \Lambda_{(\Lambda,b)}(A)$ for all $x \in f^{-1}(y)$. Hence $y \notin f[\Lambda_{(\Lambda,b)}(A)]$. Therefore $f[\Lambda_{(\Lambda,b)}(A)] \subseteq \text{Ker}(f(A))$. □

Definition 3.7 ([2]). A subset N of a space X is said to be (Λ, b)-neighborhood of a point $x \in X$ if there exists a (Λ, b)-open set U such that $x \in U \subseteq N$.

We denote the collection of all (Λ, b)-neighbourhoods of x as $\mathcal{N}_{(\Lambda,b)}(x)$.

Recall that a filter $\mathcal{F}$ on a set S is a non-empty collection of non-empty subsets of S with the properties: (a) if $F_1, F_2 \in \mathcal{F}$, then $F_1 \cap F_2 \in \mathcal{F}$, and (b) if $F \in \mathcal{F}$ and $F \subseteq G$, then $G \in \mathcal{F}$.

Definition 3.8 ([11]). Let $f : X \to Y$ be a function and $\mathcal{F}$ be a filter on X. Then the filter on Y having $f(\mathcal{F}) = \{f(A) : A \in \mathcal{F}\}$ as a base is called the image filter of $\mathcal{F}$ under f and is denoted by $f_{\sharp}(\mathcal{F})$.

Definition 3.9. A filter $\mathcal{F}$ on a space X is said to (Λ, b)-converge to $x_o \in X$ if every (Λ, b)-neighbourhood of x_o belongs to $\mathcal{F}$.

The following theorem characterizes (Λ, b)-continuous functions in terms of filter convergent.

Theorem 3.10. *A function $f : X \to Y$ is (Λ, b)-continuous at $x_o \in X$ if and only if whenever a filter $\mathcal{F}$, (Λ, b)-converges to x_o in X, then the image filter $f_{\sharp}(\mathcal{F})$ converges to $f(x_o)$ in Y.*

Proof. Assume that f is (Λ, b)-continuous at x_o and $\mathcal{F}$ is a filter (Λ, b)-converging to x_o. Let $N \in \mathcal{N}_{f(x_o)}$, the collection of all neighbourhoods of $f(x_o)$. Then there exists an open set V in Y such that $f(x_o) \in V \subseteq N$. Since f is (Λ, b)-continuous at x_o, there exists a (Λ, b)-open set U in X such that $x_o \in U$ and $f(U) \subseteq V$. By (Λ, b)-convergence of $\mathcal{F}$ to x_o in X, $U \in \mathcal{F}$. So $f(U) \in f(\mathcal{F})$. But $f(U) \subseteq N$ and so $N \in f_{\sharp}(\mathcal{F})$. It follows that $f_{\sharp}(\mathcal{F})$ converges to $f(x_o)$. Converse part: If possible, suppose that f is not (Λ, b)-continuous at x_o. Then there exists an open set V in Y containing $f(x_o)$ such that $f(U) \cap (Y \setminus V) \neq \emptyset$, for all $U \in \Lambda_b O(X, x_o)$. Now $U \cap (X \setminus f^{-1}(V)) \subseteq f^{-1}(f(U)) \cap f^{-1}(Y \setminus V) = f^{-1}(f(U) \cap (Y \setminus V)) \neq \emptyset$ implies $N \cap (X \setminus f^{-1}(V)) \neq \emptyset$ for all $N \in \mathcal{N}_{(\Lambda,b)}(x_o)$. Therefore $\mathcal{S} = \mathcal{N}_{(\Lambda,b)}(x_o) \cup \{X \setminus f^{-1}(V)\}$ has the finite intersection property and hence generates a filter, say $\mathcal{F}$ on X. Clearly $\mathcal{F}$, (Λ, b)-converges to x_o in X. Now $X \setminus f^{-1}(V) \in \mathcal{F}$ implies $f(X \setminus f^{-1}(V)) \in f(\mathcal{F})$. Since $f(X \setminus f^{-1}(V)) \subseteq Y \setminus V$, $Y \setminus V \in f_\sharp(\mathcal{F})$. Since $f_\sharp(\mathcal{F})$ is a filter, $V \notin f_\sharp(\mathcal{F})$, where V is an open neighbourhood of $f(x_o)$. Thus $f_\sharp(\mathcal{F})$ does not converge to $f(x_o)$ in Y. This contradiction proves that f is (Λ, b)-continuous at x_o. □

The following theorem represents an important characterization of (Λ, b)-irresolute function.

Theorem 3.11. *The following are equivalent for a function* $f{:}X{\to}Y$*:*

1. f *is* (Λ, b)*-irresolute;*
2. *for every* (Λ, b)*-closed subset* F *of* Y*,* $f^{-1}(F)$ *is* (Λ, b)*-closed in* X*;*
3. *for each* $x \in X$ *and for every* $V \in \Lambda_b O(Y, f(x))$*, there is a* $U \in \Lambda_b O(X, x)$ *such that* $f(U) \subseteq V$*;*
4. *for every* $A \subseteq X$*,* $f[A^{(\Lambda,b)}] \subseteq [f(A)]^{(\Lambda,b)}$*;*
5. *for every* $B \subseteq Y$*,* $[f^{-1}(B)]^{(\Lambda,b)} \subseteq f^{-1}[B^{(\Lambda,b)}]$*;*
6. *for every* $B \subseteq Y$*,* $f^{-1}[B_{(\Lambda,b)}] \subseteq [f^{-1}(B)]_{(\Lambda,b)}$.

Proof. (1) $\Longleftrightarrow$ (2): Sine $Y \setminus F \in \Lambda_b O(Y, \sigma)$ and f is (Λ, b)-irresolute, $X \setminus f^{-1}(F) = f^{-1}(Y \setminus F) \in \Lambda_b O(X, \tau)$. Hence $f^{-1}(F) \in \Lambda_b C(X, \tau)$. For converse, let $V \in \Lambda_b O(Y, \sigma)$. Then $Y \setminus V$ is (Λ, b)-closed in Y. By hypothesis, $X \setminus f^{-1}(V) = f^{-1}(Y \setminus V)$ is (Λ, b)-closed and hence $f^{-1}(V)$ is (Λ, b)-open in X. Hence f is (Λ, b)-irresolute.

(1) $\Longleftrightarrow$ (3): Let $V \in \Lambda_b O(Y, f(x))$. Then $x \in f^{-1}(V)$. Consider $U = f^{-1}(V)$. Since f is (Λ, b)-irresolute, $U \in \Lambda_b O(X, x)$ and $f(U) \subseteq V$. Conversely, suppose that $V \in \Lambda_b O(Y, \sigma)$ and $x \in f^{-1}(V)$. Then $f(x) \in V$. By assumption, there exists $U_x \in \Lambda_b O(X, x)$ such that $f(U_x) \subseteq V$. Hence $f^{-1}(V) = \bigcup\{U_x : x \in f^{-1}(V)\}$. Therefore $f^{-1}(V)$ is (Λ, b)-open in X. Hence f is (Λ, b)-irresolute.

(2) $\Longleftrightarrow$ (4): Let A be a subset of X. Then $[f(A)]^{(\Lambda,b)}$ is (Λ, b)-closed in Y and hence $f^{-1}([f(A)]^{(\Lambda,b)})$ is (Λ, b)-closed in X, by (2). Now $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}([f(A)]^{(\Lambda,b)})$ implies $A^{(\Lambda,b)} \subseteq f^{-1}([f(A)]^{(\Lambda,b)})$. So $f[A^{(\Lambda,b)}] \subseteq [f(A)]^{(\Lambda,b)}$. For converse, let F be any (Λ, b)-closed subset of Y. By assumption, we have $f([f^{-1}(F)]^{(\Lambda,b)}) \subseteq [f(f^{-1}(F))]^{(\Lambda,b)} \subseteq F^{(\Lambda,b)} = F$. Then $[f^{-1}(F)]^{(\Lambda,b)} \subseteq f^{-1}(F)$. Moreover, $f^{-1}(F) \subseteq [f^{-1}(F)]^{(\Lambda,b)}$. Thus $f^{-1}(F) = [f^{-1}(F)]^{(\Lambda,b)}$. Hence $f^{-1}(F)$ is (Λ, b)-closed in X.

(4) $\Longleftrightarrow$ (5): Let B be a subset of Y. By assumption, $f([f^{-1}(B)]^{(\Lambda,b)}) \subseteq [f(f^{-1}(B))]^{(\Lambda,b)} \subseteq B^{(\Lambda,b)}$. Hence $[f^{-1}(B)]^{(\Lambda,b)} \subseteq f^{-1}[B^{(\Lambda,b)}]$. For converse, let A be a subset of X. By assumption, $[f^{-1}(f(A))]^{(\Lambda,b)} \subseteq f^{-1}([f(A)]^{(\Lambda,b)})$. Since $A \subseteq f^{-1}(f(A))$, $A^{(\Lambda,b)} \subseteq [f^{-1}(f(A))]^{(\Lambda,b)}$. Thus $A^{(\Lambda,b)} \subseteq f^{-1}([f(A)]^{(\Lambda,b)})$ and hence $f[A^{(\Lambda,b)}] \subseteq [f(A)]^{(\Lambda,b)}$.

(1) $\Longleftrightarrow$ (6): For any $B \subseteq Y$, $B_{(\Lambda,b)}$ is (Λ, b)-open in Y and hence by (1), $f^{-1}[B_{(\Lambda,b)}]$ is (Λ, b)-open in X and is contained in $f^{-1}(B)$. Therefore

$f^{-1}[B_{(\Lambda,b)}] \subseteq [f^{-1}(B)]_{(\Lambda,b)}$. For converse, let V be (Λ, b)-open in Y. Then $V = V_{(\Lambda,b)}$ implies $f^{-1}(V) = f^{-1}[V_{(\Lambda,b)}] \subseteq [f^{-1}(V)]_{(\Lambda,b)}$, by (6). Also $[f^{-1}(V)]_{(\Lambda,b)} \subseteq f^{-1}(V)$. Thus $f^{-1}(V) = [f^{-1}(V)]_{(\Lambda,b)}$ and hence $f^{-1}(V)$ is (Λ, b)-open in X. Therefore f is (Λ, b)-irresolute. □

THEOREM 3.12. *Let $f : X \to Y$ be a (Λ, b)-irresolute function. Then for any $A \subseteq X$, $f[\Lambda_{(\Lambda,b)}(A)] \subseteq \Lambda_{(\Lambda,b)}(f(A))$.*

Proof. Assume $y \notin \Lambda_{(\Lambda,b)}(f(A))$. By Preposition 2.3, there exists a (Λ, b)-closed set V in Y such that $y \in V$ and $f(A) \cap V = \emptyset$. Then $A \cap f^{-1}(V) \subseteq f^{-1}(f(A)) \cap f^{-1}(V) = f^{-1}(f(A) \cap V) = \emptyset$ implies $A \cap f^{-1}(V) = \emptyset$. Since f is (Λ, b)-irresolute function, $f^{-1}(V)$ is (Λ, b)-closed in X. Moreover, $f^{-1}(y) \subseteq f^{-1}(V)$. Therefore, by Proposition 2.3, $x \notin \Lambda_{(\Lambda,b)}(A)$ for all $x \in f^{-1}(y)$. Hence $y \notin f[\Lambda_{(\Lambda,b)}(A)]$. Therefore $f[\Lambda_{(\Lambda,b)}(A)] \subseteq \Lambda_{(\Lambda,b)}(f(A))$. □

The next theorem characterizes (Λ, b)-irresoluteness of functions in terms of filter convergent.

THEOREM 3.13. *A function $f : X \to Y$ is (Λ, b)-irresolute at $x_o \in X$ if and only if whenever a filter $\mathcal{F}$, (Λ, b)-converges to x_o in X, then the image filter $f_\sharp(\mathcal{F})$, (Λ, b)-converges to $f(x_o)$ in Y.*

Proof. Proof is similar to Theorem 3.10. □

THEOREM 3.14. *For a function $f : X \to Y$, the following are equivalent:*

(1) *f is quasi-(Λ, b)-irresolute;*
(2) *for every b-closed subset F of Y, $f^{-1}(F)$ is (Λ, b)-closed in X;*
(3) *for every $A \subseteq X$, $f[A^{(\Lambda,b)}] \subseteq \mathrm{Cl}_b(f(A))$;*
(4) *for every $B \subseteq Y$, $[f^{-1}(B)]^{(\Lambda,b)} \subseteq f^{-1}[\mathrm{Cl}_b(B)]$;*
(5) *for every $B \subseteq Y$, $f^{-1}[\mathrm{Int}_b(B)] \subseteq [f^{-1}(B)]_{(\Lambda,b)}$.*

Proof. (1) $\iff$ (2): Since $Y \setminus V$ is b-open in Y and f is quasi-(Λ, b)-irresolute, $X \setminus f^{-1}(V) = f^{-1}(Y \setminus V)$ is (Λ, b)-open. Hence $f^{-1}(V)$ is (Λ, b)-closed in X. Conversely, let V be any b-open subset of Y. Then $Y \setminus V$ is b-closed in Y. By hypothesis, $X \setminus f^{-1}(V) = f^{-1}(Y \setminus V)$ is (Λ, b)-closed and hence $f^{-1}(V) \in \Lambda_b O(X, \tau)$. Hence f is quasi-(Λ, b)-irresolute.

(2) $\iff$ (3): Let A be a subset of X. Then $\mathrm{Cl}_b(f(A))$ is b-closed in Y and hence $f^{-1}[\mathrm{Cl}_b(f(A))]) \in \Lambda_b C(X, \tau)$, by (2). Now $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}[\mathrm{Cl}_b(f(A))]$ implies $A^{(\Lambda,b)} \subseteq f^{-1}[\mathrm{Cl}_b(f(A))]$. Hence $f[A^{(\Lambda,b)}] \subseteq \mathrm{Cl}_b(f(A))$. Conversely, let F be a b-closed subset of Y. Now , we have $f([f^{-1}(F)]^{(\Lambda,b)}) \subseteq \mathrm{Cl}_b([f(f^{-1}(F))]) \subseteq \mathrm{Cl}_b(F) = F$. Then $[f^{-1}(F)]^{(\Lambda,b)} \subseteq f^{-1}(F)$. Moreover, $f^{-1}(F) \subseteq [f^{-1}(F)]^{(\Lambda,b)}$. Thus $f^{-1}(F) = [f^{-1}(F)]^{(\Lambda,b)}$. Hence $f^{-1}(V)$ is (Λ, b)-closed in X.

(3) $\iff$ (4): Let B be a subset of Y. By (3), we have $f([f^{-1}(B)]^{(\Lambda,b)}) \subseteq \mathrm{Cl}_b([f(f^{-1}(B))]) \subseteq \mathrm{Cl}_b(B)$. Hence $[f^{-1}(B)]^{(\Lambda,b)} \subseteq f^{-1}[\mathrm{Cl}_b(B)]$. Conversely, let A be a subset of X. By (4), $[f^{-1}(f(A))]^{(\Lambda,b)} \subseteq f^{-1}[\mathrm{Cl}_b(f(A))]$. Since

$A \subseteq f^{-1}(f(A))$, $A^{(\Lambda,b)} \subseteq [f^{-1}(f(A))]^{(\Lambda,b)}$. Thus $A^{(\Lambda,b)} \subseteq f^{-1}[\mathrm{Cl}_b(f(A))]$ and hence $f[A^{(\Lambda,b)}] \subseteq \mathrm{Cl}_b(f(A))$.

(1) $\iff$ (5): For any $B \subseteq Y$, $\mathrm{Int}_b(B)$ is b-open in Y and hence by (1), $f^{-1}(\mathrm{Int}_b(B))$ is (Λ, b)-open in X and is contained in $f^{-1}(B)$. Hence $f^{-1}(\mathrm{Int}_b(B)) \subseteq [f^{-1}(B)]_{(\Lambda,b)}$. Conversely, let $V \in BO(Y, \sigma)$. Then $V = \mathrm{Int}_b(V)$ implies $f^{-1}(V) = f^{-1}(\mathrm{Int}_b(V)) \subseteq [f^{-1}(V)]_{(\Lambda,b)}$, by (5). Also we have $[f^{-1}(V)]_{(\Lambda,b)} \subseteq f^{-1}(V)$. Thus $f^{-1}(V) = [f^{-1}(V)]_{(\Lambda,b)}$ and hence $f^{-1}(V)$ is (Λ, b)-open in X. Therefore f is quasi-(Λ, b)-irresolute. □

The following is an immediate consequence of Lemma 3.2 of [3]:

LEMMA 3.15. *Let A be a subset of a space X and $x \in X$. Then $x \in b\mathrm{Ker}(A)$ if and only if $A \cap F \neq \emptyset$ for every b-closed set F containing x.*

THEOREM 3.16. *Let $f : X \to Y$ be a quasi-(Λ, b)-irresolute function. Then for every $A \subseteq X$, $f[\Lambda_{(\Lambda,b)}(A)] \subseteq b\mathrm{Ker}(f(A))$.*

Proof. Assume $y \notin b\mathrm{Ker}(f(A))$. Then there exists a b-closed set F in Y such that $y \in V$ and $f(A) \cap V = \emptyset$. Now, $A \cap f^{-1}(V) \subseteq f^{-1}(f(A)) \cap f^{-1}(V) = f^{-1}(f(A) \cap V) = \emptyset$ and its imply $A \cap f^{-1}(V) = \emptyset$. Since f is quasi-(Λ, b)-irresolute function, $f^{-1}(V)$ is (Λ, b)-closed in X. Moreover, $f^{-1}(y) \subseteq f^{-1}(V)$. Therefore, by Proposition 2.3, $x \notin \Lambda_{(\Lambda,b)}(A)$ for all $x \in f^{-1}(y)$. Hence $y \notin f[\Lambda_{(\Lambda,b)}(A)]$. Therefore $f[\Lambda_{(\Lambda,b)}(A)] \subseteq b\mathrm{Ker}(f(A))$. □

THEOREM 3.17. *Let $f : X \to Y$ be a function. Then*

(1) *f is b-continuous implies f is (Λ, b)-continuous.*
(2) *f is b-irresolute implies f is quasi-(Λ, b)-irresolute.*
(3) *f is (Λ, b)-irresolute implies f is quasi-(Λ, b)-irresolute.*
(4) *f is (Λ, b)-irresolute implies f is (Λ, b)-continuous.*

To show that converses of the results (1) and (2) of Theorem 3.17 are not true, we consider the following example.

EXAMPLE 3.18. Consider $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Then $BO(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$, $\Lambda_b O(X, \tau) = \wp(X)$, the power set of X; $BO(Y, \sigma) = \{\emptyset, \{a, \}, \{a, b\}, \{a, c\}, X\}$. Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is both (Λ, b)-continuous and quasi-(Λ, b)-irresolute but neither b-continuous nor b-irresolute. Because $V = \{a\}$ is open and hence b-open but $f^{-1}(V) = \{c\}$ is not b-open.

THEOREM 3.19. *Let $f : X \to Y$ and $g : Y \to Z$ be two functions. Then:*

(1) *If f is (Λ, b)-continuous and g is continuous, then $g \circ f : X \to Z$ is (Λ, b)-continuous.*
(2) *If f is quasi-(Λ, b)-irresolute and g is b-continuous, then $g \circ f : X \to Z$ is (Λ, b)-continuous.*
(3) *If f is (Λ, b)-irresolute and g is (Λ, b)-continuous, then $g \circ f : X \to Z$ is (Λ, b)-continuous.*

(4) *If* f *is* (Λ, b)*-irresolute and* g *is* (Λ, b)*-irresolute, then* $g \circ f : X \to Z$ *is* (Λ, b)*-irresolute.*
(5) *If* f *is quasi-*(Λ, b)*-irresolute and* g *is* b*-irresolute, then* $g \circ f : X \to Z$ *is quasi-*(Λ, b)*-irresolute.*
(6) *If* f *is* (Λ, b)*-irresolute and* g *is quasi-*(Λ, b)*-irresolute, then* $g \circ f : X \to Z$ *is quasi-*(Λ, b)*-irresolute.*

Lemma 3.20. *Let* A *be a subset of a space* X*. Then*
(1) $X \setminus A^{(\Lambda,b)} = [X \setminus A]_{(\Lambda,b)}$.
(2) $X \setminus A_{(\Lambda,b)} = [X \setminus A]^{(\Lambda,b)}$.

Proof. (1) Let $x \in X \setminus A^{(\Lambda,b)}$. Then $x \notin A^{(\Lambda,b)}$ and by Lemma 3.8 of [2], $A \cap U = \emptyset$ for some $U \in \Lambda_b O(X, x)$. Thus U is a (Λ, b)-open set contained in $X \setminus A$ and hence $U \subseteq [X \setminus A]_{(\Lambda,b)}$. Therefore $x \in [X \setminus A]_{(\Lambda,b)}$. Conversely, let $y \in [X \setminus A]_{(\Lambda,b)}$. If possible, let $y \notin X \setminus A^{(\Lambda,b)}$. Then $y \in A^{(\Lambda,b)}$ and $A \cap U \neq \emptyset$ for all $U \in \Lambda_b O(X, y)$. Since $[X \setminus A]_{(\Lambda,b)}$ is a (Λ, b)-open set containing y, $A \cap [X \setminus A]_{(\Lambda,b)} \neq \emptyset$, a contradiction.
(2) Follows from (1). □

Definition 3.21 ([2]). Let A be a subset of a space X. The (Λ, b)-frontier of A is denoted as $\Lambda_b Fr(A)$ and defined as: $\Lambda_b Fr(A) = A^{(\Lambda,b)} \cap (X \setminus A)^{(\Lambda,b)}$.

In the following theorem we use the notation $D_{(\Lambda,b)}(f)$ to stand the set of points x of X at which $f : X \to Y$ is not (Λ, b)-continuous.

Theorem 3.22. $D_{(\Lambda,b)}(f)$ *is the union of the* (Λ, b)*-frontiers of the inverse images of open sets containing* $f(x)$*.*

Proof. Let $x \in X$. Then the proof follows from the following two facts:
(i). Let f be not (Λ, b)-continuous at x. By Theorem 3.4, there exists an open set V of Y containing $f(x)$ such that $f(U) \cap (Y \setminus V) \neq \emptyset$ for all $U \in \Lambda_b O(X, x)$. Obviously $U \cap (X \setminus f^{-1}(V)) \neq \emptyset$. By Theorem 3.8 of [2], $x \in [X \setminus A]^{(\Lambda,b)}$. Also $x \in f^{-1}(V) \subseteq [f^{-1}(V)]^{(\Lambda,b)}$. Therefore $x \in \Lambda_b Fr(f^{-1}(V))$.
(ii). Let f be (Λ, b)-continuous at x. Let V be any open set of Y containing $f(x)$. Then $x \in f^{-1}(V)$, a (Λ, b)-open set of X. Then $f^{-1}(V) = [f^{-1}(V)]_{(\Lambda,b)}$, and by Lemma 3.20, $x \notin [X \setminus f^{-1}(V)]^{(\Lambda,b)}$. Hence $x \notin \Lambda_b Fr(f^{-1}(V))$. □

In Topology, homeomorphism plays an important role. We now define two important homeomorphisms via (Λ, b)-continuous and (Λ, b)-irresolute functions as weak form of homeomorphism.

Definition 3.23. A bijective function $f : (X, \tau) \to (Y, \sigma)$ is said to be Λ_b-homeomorphism (resp. $\Lambda_b r$- homeomorphism) if f and f^{-1} are (Λ, b)-continuous (resp. (Λ, b)-irresolute).

For a space (X, τ), we consider the following two important collections:
Λ_b-$h(X, \tau) = \{f \mid f : (X, \tau) \to (X, \tau) \text{ is } \Lambda_b\text{-homeomorphism}\}$;

$\Lambda_b r$-$h(X,\tau) = \{f \mid f : (X,\tau) \to (X,\tau)$ is $\Lambda_b r$-homeomorphism$\}$.
From 3.17(4), it is follows that $\Lambda_b r$-$h(X,\tau) \subseteq \Lambda_b$-$h(X,\tau)$.

THEOREM 3.24. *The collection $\Lambda_b r$-$h(X,\tau)$ forms a group under composition of functions.*

Proof. Obvious from Theorem 3.19. □

4. (Λ, b)-COMPACTNESS AND (Λ, b)-CONNECTEDNESS

In this section, we study properties of (Λ, b)-compactness and (Λ, b)-connectedness. We start by defining the notion of (Λ, b)-open cover in a space.

DEFINITION 4.1. A collection $\mathcal{A}$ of subsets of a space (X,τ) is said to be a (Λ, b)-open covering of X if the union of the elements of $\mathcal{A}$ is X and the elements of $\mathcal{A}$ are (Λ, b)-open in X.

DEFINITION 4.2. A space X is said to be (Λ, b)-compact (resp. b-compact [9]) if every (Λ, b)-open (resp. b-open) cover of X has a finite cover.

LEMMA 4.3. *Every (Λ, b)-compact space is b-compact.*

Proof. Suppose X is a (Λ, b)-compact space, and let $\mathcal{A} = \{A_\alpha : \alpha \in \Delta\}$ is a b-open cover of X. By Proposition 2.1(1), $\mathcal{A}$ is a (Λ, b)-open cover of X. Since X is (Λ, b)-compact, there is a finite subset Δ_o of Δ such that $\{A_\alpha : \alpha \in \Delta_o\}$ covers X and consequently, X is b-compact. □

COROLLARY 4.4. *Every (Λ, b)-compact space is compact.*

THEOREM 4.5. *If $f : X \to Y$ is an onto (Λ, b)-continuous function and X is (Λ, b)-compact, then Y is compact.*

Proof. Let $\{U_\alpha : \alpha \in \Delta\}$ be an open cover of Y. Since f is (Λ, b)-continuous, $\{f^{-1}(U_\alpha) : \alpha \in \Delta\}$ is a (Λ, b)-open cover of X. Since X is (Λ, b)-compact, there exists a finite subset Δ_o of Δ such that $X = \bigcup\{f^{-1}(U_\alpha) : \alpha \in \Delta_o\}$. Since f is onto, $Y = f(X) = \bigcup\{f(f^{-1}(U_\alpha)) : \alpha \in \Delta_o\} = \bigcup\{U_\alpha : \alpha \in \Delta_o\}$. Hence Y is compact. □

THEOREM 4.6. *If $f : X \to Y$ is an onto (Λ, b)-irresolute function and X is (Λ, b)-compact, then so is Y.*

Proof. Let $\{V_\alpha : \alpha \in \Delta\}$ be a (Λ, b)-open cover of Y. Since f is (Λ, b)-irresolute, $\{f^{-1}(V_\alpha) : \alpha \in \Delta\}$ is a (Λ, b)-open cover of X. Since X is (Λ, b)-compact, there exists a finite subset Δ_o of Δ such that $X = \bigcup\{f^{-1}(V_\alpha) : \alpha \in \Delta_o\}$. Since f is onto, $Y = f(X) = \bigcup\{V_\alpha : \alpha \in \Delta_o\}$. Hence Y is (Λ, b)-compact. □

THEOREM 4.7. *If $f : X \to Y$ is an onto quasi-(Λ, b)-irresolute function and X is (Λ, b)-compact, then Y is b-compact.*

Proof. Let $\{W_\alpha : \alpha \in \Delta\}$ be a b-open cover of Y. Since f is quasi-(Λ, b)-irresolute, $\{f^{-1}(W_\alpha) : \alpha \in \Delta\}$ is a (Λ, b)-open cover of X. Since X is (Λ, b)-compact, there exists a finite subset Δ_o of Δ such that $X = \bigcup\{f^{-1}(W_\alpha) : \alpha \in \Delta_o\}$. Since f is onto, $Y = f(X) = \bigcup\{W_\alpha : \alpha \in \Delta_o\}$. Hence Y is b-compact. □

DEFINITION 4.8. A space X is said to be (Λ, b)-connected (resp. b-connected [7]) if X cannot be expressed as the union of two non-empty disjoint (Λ, b)-open (resp. b-open) sets of X.

LEMMA 4.9. *Every (Λ, b)-connected space is b-connected.*

Proof. Suppose X is a (Λ, b)-connected space. If possible, let X is not b-connected. Then there exists a pair A, B of disjoint non-empty b-open subsets of X such that $X = A \cup B$. By Proposition 2.1(1), A and B are (Λ, b)-open. Therefore X is not (Λ, b)-connected, a contradiction. □

Reverse implication is considered in the following examples.

EXAMPLE 4.10. Consider the real line $\mathbb{R}$ endowed with the usual topology $\mathbb{R}_u$. Then $\mathbb{R}$ is connected but not (Λ, b)-connected because $\mathbb{Q}$, the set of rationals and $\mathbb{R} \setminus \mathbb{Q}$ together form a pair of non-empty disjoint (Λ, b)-open sets of $\mathbb{R}$ with $\mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q}) = \mathbb{R}$.

EXAMPLE 4.11. Suppose $\mathcal{F}$ is an ultrafilter on an infinite set X and $\tau = \mathcal{F} \cup \{\emptyset\}$. Then X is b-connected but not (Λ, b)-connected.

It is noticeable that there is no Hausdorff (Λ, b)-connected space.

THEOREM 4.12. *A space X is (Λ, b)-connected if and only if $A^{(\Lambda,b)} = X$ for every non-empty (Λ, b)-open subset A.*

Proof. Let X is (Λ, b)-connected. If possible, suppose A is a non-empty (Λ, b)-open subset of X such that $A^{(\Lambda,b)} \neq X$. Set $X \setminus A^{(\Lambda,b)} = B$. Then B is a non-empty (Λ, b)-open subset of X. Moreover, $A \cap B = \emptyset$. This is a contradiction. Converse part: If possible, suppose A, B is a pair of non-empty (Λ, b)-open sets of X such that $X = A \cup B$ and $A \cap B = \emptyset$. Then $A^{(\Lambda,b)} = (X \setminus B)^{(\Lambda,b)} = X \setminus B$, since $X \setminus B$ is (Λ, b)-closed. By assumption, $B = \emptyset$ which is a contradiction. □

THEOREM 4.13. *A space X is (Λ, b)-connected if and only if there is no non-empty proper subset of X which is both (Λ, b)-open and (Λ, b)-closed.*

Proof. If possible, suppose A is a non-empty proper (Λ, b)-open as well as (Λ, b)-closed subset of X. Take $B = X \setminus A$. Then $B \neq \emptyset$, B is (Λ, b)-open, $A \cap B = \emptyset$ and $A \cup B = X$. This implies X is not (Λ, b)-connected, a contradiction. Converse part: If possible, suppose $X = A \cup B$, where A and B are non-empty disjoint (Λ, b)-open subsets of X. Then $A = X \setminus B$ is (Λ, b)-closed and $A \neq X$. Thus A is a non-empty proper (Λ, b)-open as well as (Λ, b)-closed set in X. This is a contradiction. □

THEOREM 4.14. *If $f : X \to Y$ is an onto (Λ, b)-continuous function and X is (Λ, b)-connected, then Y is connected.*

Proof. If possible, suppose Y is not connected. Then there exists a pair A, B of non-empty disjoint open subsets of Y such that $Y = A \cup B$. Then $X = f^{-1}(Y) = f^{-1}(A\cup B) = f^{-1}(A)\cup f^{-1}(B)$ and $f^{-1}(A)\cap f^{-1}(B) = \emptyset$. Since f is a (Λ, b)-continuous and onto, $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty (Λ, b)-open subsets of X. Thus X is not (Λ, b)-connected. This is a contradiction. □

THEOREM 4.15. *If $f : X \to Y$ is an onto (Λ, b)-irresolute function and X is (Λ, b)-connected, then so is Y.*

THEOREM 4.16. *If $f : X \to Y$ is an onto quasi-(Λ, b)-irresolute function and X is (Λ, b)-connected, then Y is b-connected.*

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