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D.D. STANCU OPERATORS: ON SOME OF THEIR LINEAR COMBINATIONS AND GENERALIZATIONS

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Abstract. More than forty years ago, in his paper [22] from 1968, academician professor D.D. Stancu introduced and studied a new sequence of linear and positive operators, $S_n^\alpha : C[0,1] \to C[0,1]$,

$$\left(S_n^\alpha f\right)(x) = \sum_{k=0}^{n} \omega_{(n,k)}^{\alpha}(x) f\left(\frac{k}{n}\right),$$

where

$$\omega_{(n,k)}^{\alpha}(x) = \binom{n}{k} \frac{x^{[k,-\alpha]}(1-x)^{[n-k,-\alpha]}}{1^{[n,-\alpha]}},$$

$n \in \mathbb{N}$ and α is a real parameter depending only on n. We recall that $\omega_{(n,k)}^{\alpha}$ are known as "the fundamental polynomials of Stancu". This paper is concerned with linear combinations of the Stancu polynomials. The idea was inspired by O. Agratini's work from 1998 [1]. The present paper also describes other generalizations and the author summarizes various results, due to a number of authors, that are concerned with the Stancu operators.

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1. ACADEMICIAN PROFESSOR D.D. STANCU – A LIFE TIME DEDICATED TO NUMERICAL ANALYSIS AND THEORY OF APPROXIMATION

Academician Professor D.D. Stancu (1927-2014) was a Romanian distinguished mathematician, an Emeritus member of American Mathematical Society and a Honorary member of the Romanian Academy. He also was a member of the German society "Gesellschaft für Angewandte Mathematik und Mechanik". For many years he was Editor in Chief of "Revue d'Analyse Numérique et de Théorie de l'Approximation" (journal published by the Romanian Academy) and a member of the Editorial Board of the Italian mathematical journal "Calcolo", published now by Springer-Verlag, in Berlin. Also he was a reviewer of the international journal "Mathematical Reviews".

This article is a tribute dedicated to academician professor Dimitrie D. Stancu (1927 – 2014), who was my teacher, my advisor, my inspiration. "*A good teacher is like a candle. It consumes itself to light the way for others...*" (Mustafa Kemal Atatürk).

During the time, prestigious professors from "Babeş-Bolyai" University and other institutions, as professors Gheorghe Coman, Ion Păvăloiu, Petru Blaga, Octavian Agratini and many others, dedicated beautiful words to academician professor D.D. Stancu, as we can see in [3, 4, 8] and recall in the following.

Professor D.D. Stancu was born on February 11, 1927, in a farmer family, from the township Călacea, situated not far from Timişoara, the capital of Banat, a south-west province of Romania. In his school age he had many difficulties being orphan and very poor, but with the help of his mathematics teachers he succeeded to make progress in studies at the prestigious Liceum Moise Nicoară from the Arad city.

In the period 1947-1951 he studied at the Faculty of Mathematics of the University "Victor Babeş", from Cluj, Romania. When he was a student he was under the influence of professor Tiberiu Popoviciu (1906-1975), a great master of Numerical Analysis and Approximation Theory. He stimulated him to do research work and was his Ph.D. advisor. Professor D.D. Stancu obtained his Ph.D. in Mathematics in 1956.

Since 1951 he has had a continuous academic career at the "Babeş-Bolyai" University of Cluj. At the university, professor D.D. Stancu has taught several courses: Mathematical Analysis, Numerical Analysis, Approximation Theory, Informatics, Probability Theory and Constructive Theory of Functions.

The research work of academician professor D.D. Stancu included the following domains: approximation of functions by means of linear and positive operators, representation of remainders in linear approximation procedures, probabilistic methods for construction and investigation of linear positive operators, interpolation theory, spline approximation, numerical differentiation, orthogonal polynomials, numerical quadratures and cubatures, Taylor-type expansions, use of interpolation and calculus of finite differences in probability theory and mathematical statistics.

Since 1968 (in 46 years) professor D.D. Stancu has had 46 Ph.D. students from Romania, Germany and Vietnam, the author of this paper being one of his last three Ph.D. students [21].

2. D.D. STANCU OPERATORS

In his paper [22] from 1968, professor D.D. Stancu introduced and studied a new sequence of linear and positive operators, constructed using the Polya-Markov scheme

$$S_n^\alpha : C[0,1] \to C[0,1],$$

$$(S_n^\alpha f)(x) = \sum_{k=0}^{n} \omega_{(n,k)}^\alpha (x) f\left(\frac{k}{n}\right), \tag{1}$$

where

$$\omega_{(n,k)}^\alpha (x) = \binom{n}{k} \frac{x^{[k,-\alpha]}(1-x)^{[n-k,-\alpha]}}{1^{[n,-\alpha]}} \tag{2}$$

are known as "the fundamental polynomials of Stancu", $n \in \mathbb{N}$ and α is a real parameter depending only on n (see [22, 25]). The Stancu operators are very beautifully described in the books [2, 26].

Also, we mention a couple of papers. The first one [11], from 1992, belongs to B. Della Vechia, and gives a very well-informed synthesis of the principal results obtained in the theory of uniform approximation of continuous functions by means of various classes of linear positive operators of D.D. Stancu. In the second paper [12], from 1995, A. Di Lorenzo and M.R. Occorsio achieved a systematic presentation of Stancu polynomials. A brief presentation of the properties of D.D. Stancu operators was given by the present author in [21].

In 1968 [22], 1970 [23] and 1971 [24], professor D.D. Stancu proved the following properties: the operators are linear and positive, they are interpolating at the ends of the interval $[0,1]$, the norm of the operator is $||S_n^\alpha|| = 1$, for $\alpha = 0$ we find the classical Bernstein operators and for $\alpha = -\frac{1}{n}$ (2) become the Lagrange interpolation polynomials. The following identities hold true:

$$\begin{cases} \left(S_n^\alpha e_0\right)(x) = 1, \\ \left(S_n^\alpha e_1\right)(x) = x, \\ \left(S_n^\alpha e_2\right)(x) = \frac{1}{n}x + \frac{n-1}{n} \cdot \frac{x(x+\alpha)}{1+\alpha}, x \in [0,1]. \end{cases}$$

Also, professor D.D. Stancu proved the convergence theorem and gave estimations of the rate of convergence in terms of modulus of continuity. On the other hand, he showed that the operators (1) can be represented by means of the Beta function, the Bernstein operators and by finite and divided differences. Finally, he proved that the operators generate the approximation formula

$$f(x) = (S_n^\alpha f)(x) + (R_n^\alpha f)(x),$$

and gave different representations of the remainder.

In addition, other mathematicians have completed the list of properties of Stancu operators. In 1978 [18] and 1980 [20], G. Mastroianni and M.R. Occorsio discussed the variation diminishing property and the derivatives of Stancu's polynomials. In 1984 [15], H.H. Gonska and J. Meier found better constants for estimations of the rate of convergence, in terms of modulus of continuity. In 1988 [9] and 1989 [10], B. Della Vechia gave some recurrence formula and an elementary proof of the preservation of Lipschitz constants by the Stancu operators. In 2002 [17], A. Lupaş and L. Lupaş proved a representation of the remainder term and also some mean value theorems. In addition, they discussed a quadrature formula for Stancu operators. Also, in 2002 [13, 14], Z. Finta presented direct and converse results for the Stancu operators. Moreover, he proved pointwise and uniform approximation theorems using the classical moduli of smoothness and the second modulus of smoothness of Ditzian-Totik. In 2003 [5], we revealed the preservation of global smoothness of the Stancu operators, using the modulus of continuity and K-functionals.

3. LINEAR COMBINATIONS OF D.D. STANCU OPERATORS

After the pioneer work of professor D.D. Stancu, these operators have been successfully used by other mathematicians to study properties of linear positive methods of approximation.

The intensive research work and the important results obtained by professor D.D. Stancu have brought to him international recognition and appreciation. In this section we want to underline the influence of professor D.D. Stancu in other mathematicians work. Searching for "Stancu operators" on the Web of Science, it returned more than 130 results as journal articles (there are more than 60 papers containing his name in their titles).

The author chooses three linear combinations of D.D. Stancu operators: the G. Mastroianni and M.R. Occorsio generalizations from 1978, the O. Agratini linear combination from 1998 and Y. Kageyama generalization from 1999.

The reason for choosing these papers are the following: first, all three deal with the initial, one parameter, Stancu operator; secondly, all authors used different and beautiful ways to do the combinations; and finally, because they come from different parts of the world, emphasizing the fact that the influence of professor D.D. Stancu is spread all over the world.

3.1. 1978 – G. Mastroianni and M.R. Occorsio generalizations.

Based on an idea of R. Kelisky and T. Rivlin from 1967, using matrices, in [18] it was treated the following linear combination:

$$S^{\alpha}_{m,k} = I - (I - P_m^{<\alpha>})^k = \sum_{i=1}^{k} (-1)^{i-1} \binom{k}{i} (P_m^{<\alpha>})^i,$$

where I is the identity operator, which gives a better approximation than $P_m^{<\alpha>} = S^{\alpha}_{m,1}$ for sufficiently smooth functions.

First, the authors study some properties of the powers of the Stancu's operator. We recall here the generalization of the result of Kelisky and Rivlin (their outcome refers to Bernstein operators):

Theorem 1. *For all f defined on $[0,1]$ and for all $\alpha \geq 0$ we have*

$$\lim_{k\to\infty} (P_m^{<\alpha>})^k (f;x) = f(0) + [f(1) - f(0)]x.$$

Knowing that $L_m(f;x)$ are the Lagrange operators, we also have:

Theorem 2. *For all f defined on $[0,1]$ and for all $\alpha \geq 0$ we have*

$$\lim_{k\to\infty} (S^{<\alpha>}_{m,k})(f;x) = L_m(f;x), x \in [0,1].$$

Also, the authors give a generalization of the result of Voronovskaya, improving the approximation:

Theorem 3. *If $f^{(p+2k)} \in C^0[0,1], p + 2k < n, p \geq 0, k \geq 1,$ then we have*

$$\left\| D^p R^{<\alpha>}_{m,k}(f) \right\| \leq \left(\frac{\Gamma_{p+2k-2}}{m-p} \right)^k \left\| f^{(p)} \right\|_{2k},$$

where the remainder has the following form:

$$R_{m,k}^{<\alpha>} = (R_m^{<\alpha>})^k = (I - P_m^{<\alpha>})^k.$$

In the end of the paper, a convergence theorem is proved.

THEOREM 4. *For any* $f \in C^{p+2k+j}[0,1], p \geq 0, k \geq 1, 2k+j < m, j = 0,1,2,\ldots,$ *the sequence of polynomials* $\left\{D^p S_{m+p,k}^{<\alpha>}(f;x)\right\}_{m\in\mathbb{N}}$ *converges to* $f^{(p)}(x)$ *uniformly.*

This convergence property helped the authors to achieve a new numerical quadrature formula.

3.2. 1998 – O. Agratini linear combination. Based on work of R. Rathore and C. May from 1973 and 1976, in [1] it was studied another type of linear combinations

$$(D_{m,k}^{\alpha} f)(x) = \sum_{i=0}^{k} c(i,k)(P_{d_i m}^{<\alpha>} f)(x),$$

that fulfill the following conditions:

- d_i, for $i = 0,1,\ldots,k$, are arbitrary, fixed and distinct positive integers,
- $c(0,0) = 1$ and $c(i,k) = d_i^k \prod\limits_{j=0,j\neq i}^{k} (d_i - d_j)^{-1}$, $k \neq 0$,
- $\sum\limits_{i=0}^{k} c(i,k) = 1$ and $\sum\limits_{i=0}^{k} c(i,k) d_i^{-m} = 0$, $1 \leq m \leq k$.

For $d_0 = 1$ one obtains the initial Stancu operator $P_m^{<\alpha>} = D_{m,0}^{\alpha}$.

The author showed that, under definite conditions, the linear combinations approximate a function more closely than the original operator.

THEOREM 5. *The following identities hold:*

$$\begin{cases} D_{m,k}^{\alpha} e_0 = e_0, \\ D_{m,k}^{\alpha} e_1 = e_1, \\ D_{m,k}^{\alpha} e_2 = \frac{\alpha}{\alpha+1} e_1 + \frac{1}{\alpha+1} e_2. \end{cases}$$

Using this and the s-th order central moments of the operator $P_m^{<\alpha>}$:

$$\mu_{(m,s)}(\alpha;x) = (P_m^{<\alpha>} \varphi_x^s)(x), \varphi_x = e_1 - x e_0,$$

the author proved the following

THEOREM 6. *If* $0 < \alpha(m) \to 0$ *and* $m \to \infty$, f *is bounded on* $[0,1]$ *and* $2k+2$ *times differentiable at some point* $x \in [0,1]$, *then*

$$|(D_{m,k}^{\alpha} f)(x) - f(x)| \leq C_k m^{-(k+1)},$$

where C_k *is a constant that depends on* k.

3.3. 1999 – Y. Kageyama generalizations. In [16] it was introduced another specific class of operators generated from the D.D. Stancu operators, having stability and convergence rate properties similar to those of Sablonniere's operators

$$({}_{\alpha}B_n f)(x) = \sum_{j=0}^{\alpha} \frac{(-1)^j}{n^j j!} \frac{\partial^j P_n^{<s>} f(x)}{\partial s^j}|_{s=0}.$$

This is generated by putting $s = -1/n$ in the MacLaurin series truncated at degree α of $P_n^{<s>} f(x)$ regarded as a function of s. The new operators have the following explicit representation:

THEOREM 7.

$$({}_{\alpha}B_n f)(x) = \sum_{\nu=0}^{n} f\left(\frac{\nu}{n}\right) \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} \binom{kx}{\nu} \binom{k(1-x)}{n-\nu} \left(\frac{k}{n}\right)^{\alpha},$$

where $f : [0,1] \to \mathbb{R}$ *and* $x \in [0,1]$.

The following theorem concerning stability and convergence rate represents the most important result in the paper of Kageyama.

THEOREM 8. *For each* $\alpha \in \mathbb{N}_0$*, the sequence* $\{{}_{\alpha}B_n\}_{n=1}^{\infty}$ *has the following properties:*

(1) *For all* $p, q, r \in \mathbb{N}_0$*, there is* M *such that for all* $n \in \mathbb{N}$ *and for all* $f \in C^r[0,1]$*, we have*

$$\left\| [x(1-x)]^p ({}_{\alpha}B_n f)^{(q+r)} \right\| \leq M n^{q-\min\{p,[q/2]\}} \left\| f^{(r)} \right\|.$$

(2) *For all* $\beta, \gamma \in \mathbb{N}_0$*,* $\beta \leq \alpha$ *and for all* $f \in C^{2\beta+\gamma}[0,1]$*, we have*

$$\left\| ({}_{\alpha}B_n f)^{(\gamma)} - f^{(\gamma)} \right\| = o(n^{-\beta}), n \to \infty.$$

(3) *For all* $\gamma \in \mathbb{N}_0$ *and for all* $f \in C^{2\alpha+\gamma+2}[0,1]$ *we have*

$$\lim_{k\to\infty} n^{\alpha+1} \left(({}_{\alpha}B_n f)^{(\gamma)} - f^{(\gamma)} \right) = -\left(\sum_{k=0}^{2\alpha+2} \Upsilon_{\alpha+1,k} \frac{f^{(k)}}{k!} \right)^{(\gamma)}.$$

Also, the author gives an application to numerical quadrature, and he collects several propositions about Stancu operator.

4. GENERALIZATIONS OF D.D. STANCU OPERATORS

There are numerous combinations and generalizations of Stancu operators: Bernstein-Stancu operators, Schurer-Stancu operators, Durrmeyer-Stancu operators, Baskakov-Stancu operators, Stancu-Hurwitz operators, Stancu-Muhlbach operators, Stancu-Beta operators, Stancu-Meyer, Koning and Zeller operator, Kantorovich-Stancu operators, Bernstein-Schurer-Stancu operators, Baskakov-Durrmeyer-Stancu operators, Baskakov-Beta-Stancu operators and many others. Some of them are made in the new direction of q-calculus.

We draw a diagram of most of these new operators, created using the Stancu operator (with one or two parameters).

In the following, we recall our generalization from 2007.

4.1. **2007 – Bernstein-Stancu operators (V.A. Cleciu (Radu)).** In [6] it was introduced and investigated the modifications operators $C_n : Y \to \Pi_n$,

$$(C_n f)(x) = \sum_{k=0}^{n} \frac{k!}{n^k} \binom{n}{k} \mathbf{m}_{k,n} \left[0, \frac{1}{n}, \ldots, \frac{k}{n}; f\right] x^k, \ \ f \in Y,$$

where the real numbers $(m_{k,n})_{k=0}^{\infty}$ are selected in order to preserve some important properties of Bernstein operators.

For $\mathbf{m}_{j,n} = \dfrac{(a_n)_j}{j!}, a_n \in (0,1]$ we obtain

$$\left(\overline{C}_n f\right)(x) = \sum_{k=0}^{n} \frac{(a_n)_k}{n^k} \binom{n}{k} \left[0, \frac{1}{n}, \ldots, \frac{k}{n}; f\right] x^k, \ \ f \in Y.$$

For $g : [0,1] \to \mathbb{R}$ we recall this form of Stancu operators $S_k : g \to S_k g$, $k \in \mathbb{N}$, defined as $\left(S_0^{<b>} g\right)(x) = g(0)$ and for $k \in \{1, 2, \ldots\}$:

$$\left(S_k^{<b>} g\right)(x) = \frac{1}{(b)_k} \sum_{j=0}^{k} \binom{k}{j} (bx)_j \, (b - bx)_{k-j} \, g\left(\frac{j}{k}\right), \quad x \in [0,1],$$

where $b \in [0,1]$ is a parameter. Observe that $\overline{C}_0 f = \overline{C}_{0,0}[f] := f(0)$ and

$$\left(S_k^{<1>} g\right)(a_n) = \frac{1}{k!} \sum_{j=0}^{k} \binom{k}{j} (a_n)_j \,(1-a_n)_{k-j}\, g\left(\frac{j}{k} \cdot \frac{k}{n}\right),\ k \geq 1.$$

Therefore, $\overline{C}_{k,n}[f] = \left(S_k^{<1>} g_{n,k}^{<f>}\right)(a_n)$ with $g_{n,k}^{<f>}(t) = f\left(t\frac{k}{n}\right)$, $k \geq 1$.

Definition 9. The linear transformations $\overline{C}_{k,n} : Y \to \mathbb{R}$, $k \in \{0,1,2,\ldots,n\}$, $n \in \mathbb{N}^*$ are Stancu functionals. When $a_n \in (0,1)$, the linear positive transformations $\overline{C}_n : Y \to \Pi_n$, $n \in \mathbb{N}^*$, are called **Bernstein-Stancu operators**.

Lemma 10. *The following identities hold:*

$$\begin{cases} \left(\overline{C}_n e_0\right)(x) = 1, \\ \left(\overline{C}_n e_1\right)(x) = a_n x = x - (1-a_n)\,x, \\ \left(\overline{C}_n e_2\right)(x) = x^2 + \dfrac{x(1-x)}{n} a_n + \dfrac{1-a_n}{2}\left(\frac{a_n}{n} - (2+a_n)\right) x^2. \end{cases}$$

The theorem of convergence is established in [7].

Theorem 11. *The sequence* $\{\overline{C}_n f\ \}_{n\geq 1}$ *converges to* f*, uniformly on* $[0,1]$ *for any* $f \in Y$*.*

Also, some estimations of the rate of convergence in terms of modulus of continuity are given.

In conclusion, positive linear operators play an important role in approximation theory. Throughout this article, we wanted to emphasize the fact that, in the development of the theory of approximation by positive and linear operators, the Romanian mathematicians have brought very important contributions, especially academician professor D.D. Stancu. Other remarkable Romanian mathematicians in this field are: O. Agratini, P. Blaga, Gh. Coman, I. Gavrea, M. Ivan, A. Lupaş, R. Păltănea, I. Raşa, R. Trâmbiţaş and others.

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