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ON T-OPEN SETS AND SEMI-COMPACT SPACES

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Abstract. In this paper, we introduce and investigate a new class of sets called T-open sets which are weaker semi-open sets. Moreover, we obtain characterizations and preserving theorems of semi-compact spaces.

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Key words. semi-open set, semi-compact space, S-closed space.

1. INTRODUCTION

Throughout this paper, (X, τ) and (Y, σ) stand for topological spaces on which no separation axiom is assumed, unless otherwise stated. For a subset A of X, the closure of A and the interior of A will be denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. Let (X, τ) be a space and S a subset of X. A subset S of X is said to be semi-open [9] if there exists an open set U of X such that $U \subseteq S \subseteq \mathrm{Cl}(U)$, or equivalently if $S \subseteq \mathrm{Cl}(\mathrm{Int}(S))$. The complement of a semi-open set is said to be semi-closed. The intersection of all semi-closed sets containing S is called the semi-closure of S and is denoted by $\mathrm{sCl}(S)$. The semi-interior of S, denoted by $\mathrm{sInt}(S)$, is defined by the union of all semi-open sets contained in S. It is verified in [3] that $\mathrm{sCl}(A) = A \cup \mathrm{Int}(\mathrm{Cl}(A))$ and $\mathrm{sInt}(A) = A \cap \mathrm{Cl}(\mathrm{Int}(A))$ for any subset $A \subseteq X$. The family of all semi-open sets of X is denoted by $SO(X)$. Moreover, for each $x \in X$ the family $\{U \in SO(X) \mid x \in U\}$ is denoted by $SO(X, x)$.

2. T-OPEN SETS

In this section we introduce the following notion:

Definition 1. A subset A of a space X is said to be T-open if for every $x \in A$, there exists a semi-open subset $U_x \subseteq X$ containing x such that $U_x - A$ is finite. The complement of a T-open subset is said to be T-closed.

The family of all T-open subsets of a space (X, τ) is denoted by $TO(X)$. For each $x \in X$, $TO(X, x)$ denotes the family $\{U \in TO(X) : x \in U\}$.

Definition 2. A subset A of a space X is said to be:

(1) α-open [10] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$;
(2) β-open [1] or semi-preopen [3] if $A \subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$;

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(3) b-open [2] if $A \subseteq \mathrm{Cl}(\mathrm{Int}(A)) \cup \mathrm{Int}(\mathrm{Cl}(A))$.

The family of all α-open (resp. b-open, β-open) subsets of a space (X, τ) is denoted by $\alpha O(X)$ (resp. $BO(X)$, $SPO(X)$).

LEMMA 1. *For a subset of a topological space, semi-openness imply T-openness.*

Proof. Let A be semi-open. For each $x \in A$, there exists a semi-open set $U_x = A$ such that $x \in U_x$ and $U_x - A = \varnothing$. Therefore, A is T-open. □

For a subset of a topological space, the following implications hold:

$$\begin{array}{ccccccc} & & T\text{-open} & & & & \\ & & \uparrow & & & & \\ \text{open} & \longrightarrow & \text{semi-open} & \longrightarrow & b\text{-open} & \longrightarrow & \beta\text{-open} \end{array}$$

The converses need not be true as shown by the following examples.

EXAMPLE 1. The digital line or so called *Khaimsky* line is the set of the integers, $\mathcal{Z}$ equipped with the topology κ having $\{\{2n-1, 2n, 2n+1\} | n \in \mathcal{Z}\}$ as a subbase. It is denoted by $(\mathcal{Z}, \kappa)$. A singleton $\{2n+1\}$ is open and a subset $\{2n-1, 2n, 2n+1\}$ is the smallest open set containing $2n$, where n is any integer. Let $a \in \mathcal{Z}$ be an even number, then $U_a = \{a\}$ is a T-open set which is not semi-open. (Since for each $x \in U_a$, there is a semi-open set $V_a = \{a, a+1, a+2\}$ such that $V_a - U_a$ is finite and $U_a \nsubseteq \mathrm{Cl}(\mathrm{Int}(U_a)) = \varnothing$) i.e. in the digital line every singleton is T-open. Also by Theorem 1 (below) we have $U_a = \{a\}$ is T-open set but neither b-open nor β-open.

THEOREM 1. [12] *For $(\mathcal{Z}, \kappa)$ we have the following property:*

$$SO(\mathcal{Z}, \kappa) = BO(\mathcal{Z}, \kappa) = SPO(\mathcal{Z}, \kappa).$$

EXAMPLE 2. Consider the set $\mathbb{R}$ of real numbers with the usual topology, and let $A = [0,1] \cup ((1,2) \cap \mathbb{Q})$, where $\mathbb{Q}$ stands for the set of rational numbers. Then A is b-open but which is not T-open, since any semi-open set U containing $\frac{3}{2}$, $U \setminus A$ is infinite.

THEOREM 2. *Let (X, τ) be a topological space. Then the arbitrary union of T-open sets of X is T-open.*

Proof. Let $\{U_i : i \in I\}$ be a family of T-open subsets of X and $x \in \cup_{i \in I} U_i$. Then $x \in U_j$ for some $j \in I$. This implies that there exists a semi-open subset V of X containing x such that $V \setminus U_j$ is finite. Since $V \setminus \cup_{i \in I} U_i \subseteq V \setminus U_j$, then $V \setminus \cup_{i \in I} U_i$ is finite. Thus $\cup_{i \in I} U_i \in TO(X)$. □

The following example shows that the intersection of two T-open sets need not be T-open.

EXAMPLE 3. Let X be the usual space of reals, then $A = (0,1]$, $B = [1,3)$ are semi-open sets and T-open sets but $A \cap B = \{1\}$ is not T-open in X.

LEMMA 2. [10] *Let (X, τ) be a topological space. Then the intersection of an α-open set and a semi-open set is semi-open.*

LEMMA 3. *Let (X, τ) be a topological space. Then the intersection of an α-open set and a T-open set is T-open.*

Proof. Let U be α-open and A be T-open. Then for every $x \in A$, there exists a semi-open set $V_x \subseteq X$ containing x such that $V_x - A$ is finite, also by Lemma 2, $U \cap V_x$ is semi-open. Now for each $x \in U \cap A$, there exists a semi-open set $U \cap V_x \subseteq X$ containing x and

$$\begin{aligned}(U \cap V_x) - (U \cap A) =& (U \cap V_x) \cap [(X - U) \cup (X - A)] \\ =& [(U \cap V_x) \cap (X - U)] \cup [(U \cap V_x) \cap (X - A)] \\ =& (U \cap V_x) - A \\ \subseteq& V_x - A.\end{aligned}$$

Then $(U \cap V_x) - (U \cap A)$ is finite. Therefore $U \cap A$ is a T-open set. □

LEMMA 4. *A subset A of a space X is T-open if and only if for every $x \in A$, there exist a semi-open subset U containing x and a finite subset C such that $U - C \subseteq A$.*

Proof. Let A be T-open and $x \in A$, then there exists a semi-open subset U_x containing x such that $U_x - A$ is finite. Let $C = U_x - A = U_x \cap (X - A)$. Then $U_x - C \subseteq A$. Conversely, let $x \in A$. Then there exist a semi-open subset U_x containing x and a finite subset C such that $U_x - C \subseteq A$. Thus $U_x - A \subseteq C$ and $U_x - A$ is a finite set. □

THEOREM 3. *Let X be a space and $C \subseteq X$. If C is T-closed, then $C \subseteq K \cup B$ for some semi-closed subset K and a finite subset B.*

Proof. If C is T-closed, then $X - C$ is T-open and hence for every $x \in X - C$, there exist a semi-open set U containing x and a finite set B such that $U - B \subseteq X - C$. Thus $C \subseteq X - (U - B) = X - (U \cap (X - B)) = (X - U) \cup B$. Let $K = X - U$. Then K is a semi-closed such set that $C \subseteq K \cup B$. □

DEFINITION 3. [5] A function $f : X \to Y$ is said to be pre-semi-open if $f(V)$ semi-open in Y for each semi-open set V in X.

PROPOSITION 1. If $f : X \to Y$ is pre-semi-open, then the image of a T-open set of X is T-open in Y.

Proof. Let $f : X \to Y$ be pre-semi-open and W a T-open subset of X. Let $y \in f(W)$, there exists $x \in W$ such that $f(x) = y$. Since W is T-open, there exists a semi-open set U such that $x \in U$ and $U - W = C$ is finite. Since f is pre-semi-open, $f(U)$ is semi-open in Y such that $y = f(x) \in f(U)$ and $f(U) - f(W) \subseteq f(U - W) = f(C)$ is finite. Hence $f(W)$ is T-open in Y. □

DEFINITION 4. [5] A function $f : X \to Y$ is said to be irresolute if $f^{-1}(V)$ is semi-open in X for each semi-open set V in Y.

PROPOSITION 2. If $f : X \to Y$ is irresolute injective and A is T-open in Y, then $f^{-1}(A)$ is T-open in X.

Proof. Assume that A is a T-open subset of Y. Let $x \in f^{-1}(A)$. Then $f(x) \in A$ and there exists a semi-open set V containing $f(x)$ such that $V - A$ is finite. Since f is irresolute, $f^{-1}(V)$ is a semi-open set containing x. Thus $f^{-1}(V) - f^{-1}(A) = f^{-1}(V - A) =$ finite. Then $f^{-1}(A)$ is T-open in X. □

3. SEMI-COMPACT SPACES

DEFINITION 5. (1)[4] A space X is said to be semi-compact if every semi-open cover of X has a finite subcover.

(2) A subset A of a space X is said to be semi-compact relative to X if every cover of A by semi-open sets of X has a finite subcover.

THEOREM 4. *If X is a space such that every semi-open subset of X is semi-compact relative to X, then every subset of X is semi-compact relative to X.*

Proof. Let B be an arbitrary subset of X and let $\{U_i : i \in I\}$ be a cover of B by semi-open sets of X. Then the family $\{U_i : i \in I\}$ is a semi-open cover of the semi-open set $\cup\{U_i : i \in I\}$. Hence by hypothesis there is a finite subfamily $\{U_{i_j} : j \in \mathbb{N}_0\}$ which covers $\cup\{U_i : i \in I\}$ where $\mathbb{N}_0$ is a finite subset of the naturals $\mathbb{N}$. This subfamily is also a cover of the set B. □

THEOREM 5. *For any space X, the following properties are equivalent:*

(1) *X is semi-compact;*
(2) *Every T-open cover of X has a finite subcover.*

Proof. (1) ⇒ (2): Let $\{U_\alpha : \alpha \in \Lambda\}$ be any T-open cover of X. For each $x \in X$, there exists $\alpha(x) \in \Lambda$ such that $x \in U_{\alpha(x)}$. Since $U_{\alpha(x)}$ is T-open, there exists a semi-open set $V_{\alpha(x)}$ such that $x \in V_{\alpha(x)}$ and $V_{\alpha(x)} \setminus U_{\alpha(x)}$ is finite. The family $\{V_{\alpha(x)} | x \in X\}$ is a semi-open cover of X and X is semi-compact. There exists a finite subset, says $\alpha(x_1), \alpha(x_2), \cdots, \alpha(x_n)$ such that $X = \cup\{V_{\alpha(x_i)} | i \in F = \{1, 2, \ldots, n\}\}$. Now, we have

$$X = \cup_{i \in F} \{(V_{\alpha(x_i)} \setminus U_{\alpha(x_i)}) \cup U_{\alpha(x_i)}\}$$
$$= [\cup_{i \in F}(V_{\alpha(x_i)} \setminus U_{\alpha(x_i)})] \cup [\cup_{i \in F} U_{\alpha(x_i)}].$$

For each $\alpha(x_i)$, $V_{\alpha(x_i)} \setminus U_{\alpha(x_i)}$ is a finite set and there exists a finite subset $\Lambda_{\alpha(x_i)}$ of Λ such that $V_{\alpha(x_i)} \setminus U_{\alpha(x_i)} \subseteq \cup\{U_\alpha | \alpha \in \Lambda_{\alpha(x_i)}\}$. Therefore, we have $X \subseteq [\cup_{i \in F}(\cup\{U_\alpha | \alpha \in \Lambda_{\alpha(x_i)}\})] \cup [\cup_{i \in F} U_{\alpha(x_i)}]$.

(2) ⇒ (1): Since every semi-open is T-open, the proof is obvious. □

THEOREM 6. *For any space X, the following properties are equivalent:*

(1) *X is semi-compact;*
(2) *Every proper T-closed set is semi-compact with respect to X.*

Proof. (1) $\Rightarrow$ (2): Let A be a proper T-closed subset of X. Let $\{U_\alpha : \alpha \in \Lambda\}$ be a cover of A by semi-open sets of X. Now for each $x \in X - A$, there is a semi-open set V_x such that $V_x \cap A$ is finite. Since $\{U_\alpha : \alpha \in \Lambda\} \cup \{V_x : x \in X - A\}$ is a semi-open cover of X and X is semi-compact, there exists a finite subcover $\{U_{\alpha_i} : i \in F_1 = \{1, 2, \ldots, n\}\} \cup \{V_{x_i} : i \in F_2 = \{1, 2, \ldots, m\}\}$. Since $\cup_{i \in F_2}(V_{x_i} \cap A)$ is finite, so for each $x_j \in \cup_{i \in F_2}(V_{x_i} \cap A)$, there is $U_{\alpha(x_j)} \in \{U_\alpha : \alpha \in \Lambda\}$ such that $x_j \in U_{\alpha(x_j)}$ and $j \in F_3$ where F_3 is finite . Hence $\{U_{\alpha_i} : i \in F_1\} \cup \{U_{\alpha(x_j)} : j \in F_3\}$ is a finite subcover of $\{U_\alpha : \alpha \in \Lambda\}$ and it covers A. Therefore, A is semi-compact relative to X.

(2) $\Rightarrow$ (1): Let $\{V_\alpha : \alpha \in \Lambda\}$ be any semi-open cover of X. We choose and fix one $\alpha_0 \in \Lambda$. Then $X - V_{\alpha_0} \subset \cup\{V_\alpha : \alpha \in \Lambda - \{\alpha_0\}\}$ is a semi-open cover of a T-closed set $X - V_{\alpha_0}$. There exists a finite subset Λ_0 of $\Lambda - \{\alpha_0\}$ such that $X - V_{\alpha_0} \subset \cup\{V_\alpha : \alpha \in \Lambda_0\}$. Therefore, $X = \cup\{V_\alpha : \alpha \in \Lambda_0 \cup \{\alpha_0\}\}$. This shows that X is semi-compact. □

Corollary 1. *If a space X is semi-compact and A is semi-closed, then A is semi-compact relative to X.*

Definition 6. A function $f : X \to Y$ is said to be T-continuous if $f^{-1}(V)$ is T-open in X for each open set V in Y.

Theorem 7. *A function $f : X \to Y$ is T-continuous if and only if for each point $x \in X$ and each open set V in Y with $f(x) \in V$, there is a T-open set U in X such that $x \in U$ and $f(U) \subseteq V$.*

Proof. Sufficiency. Let V be open in Y and let $x \in f^{-1}(V)$. Then $f(x) \in V$ and thus there exists an $U_x \in TO(X)$ such that $x \in U_x$ and $f(U_x) \subseteq V$. Then $x \in U_x \subseteq f^{-1}(V)$ and $f^{-1}(V) = \cup_{x \in f^{-1}(V)} U_x$. Then by Theorem 2 $f^{-1}(V)$ is T-open.

Necessity . Let $f(x) \in V$. Then $x \in f^{-1}(V) \in TO(X)$ since $f : X \to Y$ is T-continuous. Let $U = f^{-1}(V)$. Then $x \in U$ and $f(U) \subseteq V$. □

Theorem 8. *Let f be a T-continuous function from a space X onto a space Y. If X is semi-compact, then Y is compact.*

Proof. Let $\{V_\alpha : \alpha \in \Lambda\}$ be an open cover of Y. Then $\{f^{-1}(V_\alpha) : \alpha \in \Lambda\}$ is a T-open cover of X. Since X is semi-compact, by Theorem 5, there exists a finite subset Λ_0 of Λ such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in \Lambda_0\}$; hence $Y = \cup\{V_\alpha : \alpha \in \Lambda_0\}$. Therefore Y is compact. □

Definition 7. [9] A function $f : X \to Y$ is said to be semi-continuous if $f^{-1}(V)$ is semi-open for each open set V in Y.

Remark 1. It is shown in Example 4 (below) that every semi-continuous function is T-continuous but not conversely.

Corollary 2. [4] *Let f be a semi-continuous function from a space X onto a space Y. If X is semi-compact, then Y is compact.*

Definition 8. A function $f : X \to Y$ is said to be sT-continuous if $f^{-1}(V)$ is T-open in X for each semi-open set V in Y.

It is clear that a function $f : X \to Y$ is sT-continuous if and only if for each point $x \in X$ and each semi-open set V in Y with $f(x) \in V$, there is a T-open set U in X such that $x \in U$ and $f(U) \subseteq V$.

Definition 9. A function $f : X \to Y$ is said to be weakly sT-continuous if for each $x \in X$ and each semi-open set V of Y containing $f(x)$, there exists $U \in TO(X, x)$ such that $f(U) \subseteq \mathrm{Cl}(V)$.

Definition 10. [6] A function $f : X \to Y$ is said to be weakly θ-irresolute if for each $x \in X$ and each semi-open set V of Y containing $f(x)$, there exists $U \in SO(X, x)$ such that $f(U) \subseteq \mathrm{Cl}(V)$.

The following implications hold:

$$\begin{array}{ccccc}
\text{semi-continuous} & \longleftarrow & \text{irresolute} & \longrightarrow & \text{weakly } \theta\text{-irresolute} \\
\downarrow & & \downarrow & & \downarrow \\
T\text{-continuous} & \longleftarrow & sT\text{-continuous} & \longrightarrow & \text{weakly } sT\text{-continuous}
\end{array}$$

The following examples show that:

(1) Semi-continuity and sT-continuity are independent of each other.
(2) Weakly θ-irresoluteness and sT-continuity are independent of each other.

Therefore, all implications in this diagram are not reversible.

Example 4. Let $X = \{a, b, c\}$, $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\}$ and $f : (X, \tau) \to (X, \tau)$ be the function defined by setting $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is sT-continuous but not semi-continuous.

Example 5. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\}$ and σ=$\{\varnothing, Y, \{c\}\}$. Let $f : (Y, \sigma) \to (X, \tau)$ be the function defined by setting $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is sT-continuous but not weakly θ-irresolute [[6], Theorem 1.2], since $\{b, c\}$ is regular closed in X and $f^{-1}(\{b, c\}) = \{a, b\} \notin SO(Y)$.

Example 6. Let X be an infinite set and let A, B and C be subsets of X such that each of them is infinite and the family $\{A, B, C\}$ is a partition of X.

(1) We define the topology τ=$\{\varnothing, X, \{A\}, \{B, C\}\}$ and σ=$\{\varnothing, X, \{A\}\}$. Let $f : (X, \tau) \to (X, \sigma)$ be the function defined by setting $f(\{A\}) = \{A\}$, $f(\{B\}) = \{C\}$ and $f(\{C\}) = \{B\}$. Then f is semi-continuous but not sT-continuous, since $f^{-1}(\{A, B\}) = \{A, C\}$ which is not a T-open set.
(2) We define the topology τ=$\{\varnothing, X, \{C\}\}$ and σ=$\{\varnothing, X, \{A\}, \{B\}, \{A, B\}\}$. If $f : (X, \tau) \to (X, \sigma)$ is the identity function, then f is weakly θ-irresolute but not sT-continuous, since $f^{-1}(\{A\}) = \{A\}$ which is not a T-open set.

Now we state the following theorem whose proof is similar to Theorem 8.

Theorem 9. *Let f be an sT-continuous function from a space X onto a space Y. If X is semi-compact, then Y is semi-compact.*

Corollary 3. [8] *If $f : X \to Y$ is an irresolute surjection and X is semi-compact, then Y is semi-compact.*

Definition 11. [13] A topological space X is said to be S-closed if for every semi-open cover $\{U_\alpha : \alpha \in \Lambda\}$ of X there exists a finite subset $\Lambda_0 \subseteq \Lambda$ such that $X = \cup\{\mathrm{Cl}(U_\alpha) : \alpha \in \Lambda_0\}$.

Theorem 10. *Let $f : X \to Y$ be a weakly sT-continuous surjection. If X is semi-compact, then Y is S-closed.*

Proof. Let $\{V_\alpha : \alpha \in \Lambda\}$ be a semi-open cover of Y. For each $x \in X$, there exists $\alpha(x) \in \Lambda$ such that $f(x) \in V_{\alpha(x)}$. Since f is weakly sT-continuous, there exists a T-open set $U_{\alpha(x)}$ of X containing x such that $f(U_{\alpha(x)}) \subseteq \mathrm{Cl}(V_{\alpha(x)})$. Now $\{U_{\alpha(x)} : x \in X\}$ is a T-open cover of the semi-compact space X. So by Theorem 5 there exist a finite numbers of points, says, $x_1, x_2, \ldots, x_n$ in X such that $X = \cup\{U_{\alpha(x_i)} : 1 \le i \le n\}$. Thus $Y = f(\cup\{U_{\alpha(x_i)} : 1 \le i \le n\}) = \cup\{f(U_{\alpha(x_i)}) : 1 \le i \le n\}) \subseteq \cup\{\mathrm{Cl}(V_{\alpha(x_i)}) : 1 \le i \le n\}$. This shows that Y is S-closed. $\square$

Corollary 4. [6] *If $f : X \to Y$ is a weakly θ-irresolute surjection and X is semi-compact, then Y is S-closed.*

Definition 12. A function $f : X \to Y$ is said to be sT-closed (resp. T-closed) if $f(A)$ is T-closed in Y for each semi-closed (resp. closed) set A of X.

Definition 13. A function $f : X \to Y$ is said to be presemiclosed [7] (resp. semi-closed [11]) if $f(F)$ is semi-closed in Y for each semi-closed (resp. closed) set F of X.

The following implications hold:

$$\begin{array}{ccccc} & & \text{pre-semi-closed} & \longrightarrow & sT\text{-closed} \\ & & \downarrow & & \downarrow \\ \text{closed} & \longrightarrow & \text{semi-closed} & \longrightarrow & T\text{-closed} \end{array}$$

The following examples show that semi-closedness and sT-closedness are independent of each other.

Example 7. Let $X = Y = \{a, b, c\}$, $\tau = \{X, \varnothing, \{a\}, \{b\}, \{a, b\}\}$ and σ=$\{\varnothing, Y, \{c\}\}$. If $f : (Y, \sigma) \to (X, \tau)$ is the identity function. Then f is sT-closed but not semi-closed, since $\{a, b\}$ is closed in Y and $f(\{a, b\}) = \{a, b\}$ is not semi-closed in X.

Example 8. Let $Y = \{a, b, c, d\}$, $\sigma = \{Y, \varnothing, \{a\}, \{a, b, d\}\}$, and (X, τ) be the same topological space as (1) in Example 6. If $f : (Y, \sigma) \to (X, \tau)$ the function define by setting $f(\{a\}) = \{B\}$, $f(\{b\}) = \{B\}$, $f(\{c\}) = \{A\}$ and $f(\{d\}) = \{C\}$. Then f is semi-closed but not sT-closed, since $f(\{b\}) = \{B\}$ which is not T-closed.

Theorem 11. *If $f : X \to Y$ is an sT-closed surjection such that $f^{-1}(y)$ is semi-compact relative to X for each $y \in Y$ and Y is semi-compact, then X is semi-compact.*

Proof. Let $\{U_\alpha : \alpha \in \Lambda\}$ be any semi-open cover of X. For each $y \in Y$, $f^{-1}(y)$ is semi-compact relative to X and there exists a finite subset $\Lambda(y)$ of Λ such that $f^{-1}(y) \subset \cup\{U_\alpha : \alpha \in \Lambda(y)\}$. Now we put $U(y) = \cup\{U_\alpha : \alpha \in \Lambda(y)\}$ and $V(y) = Y - f(X - U(y))$. Then, since f is sT-closed, $V(y)$ is a T-open set in Y containing y such that $f^{-1}(V(y)) \subset U(y)$. Since $\{V(y) : y \in Y\}$ is a T-open cover of Y, by Theorem 5 there exist a finite numbers of points, says, $y_1, y_2, \ldots, y_n$ in Y such that $Y = \cup\{V(y_i) : 1 \leq i \leq n\}$. Therefore, $X = f^{-1}(Y) = \cup\{f^{-1}(V(y_i)) : 1 \leq i \leq n\} \subseteq \cup\{U(y_i) : 1 \leq i \leq n\} = \cup\{U_\alpha : \alpha \in \Lambda(y_i), 1 \leq i \leq n\}$. This shows that X is semi-compact. □

Corollary 5. *If $f : X \to Y$ is a presemiclosed surjection such that $f^{-1}(y)$ is semi-compact with respect to X for each $y \in Y$ and Y is semi-compact, then X is semi-compact.*

Corollary 6. *Let $f : X \to Y$ be an sT-closed and sT-continuous surjection such that $f^{-1}(y)$ is semi-compact relative to X for each $y \in Y$. Then X is semi-compact if and only if Y is semi-compact.*

Proof. This is an immediate consequence of Theorems 9 and 11. □

Theorem 12. *If $f : X \to Y$ is a T-closed surjection such that $f^{-1}(y)$ is compact in X for each $y \in Y$ and Y is semi-compact, then X is compact.*

Proof. The proof is analogous to that of Theorem 11. □

Corollary 7. *If $f : X \to Y$ is a semi-closed surjection such that $f^{-1}(y)$ is compact in X for each $y \in Y$ and Y is semi-compact, then X is compact.*

REFERENCES

[1] Abd El-Monsef, M.E., El-Deeb, S.N. and Mahmoud, R.A., *β-open sets and β-continuous mappings*, Bull. Fac. Sci. Assiut Univ. A, **12** (1983), 77–90.

[2] Andrijević, D. , *On b-open sets*, Mat. Vesnik, **48** (1996), 59–64.

[3] Andrijević, D., *Semi-preopen sets*, Mat. Vesnik, **38** (1986), 24–32.

[4] Carnahan, D. A., *Some properties related to compactness in topological spaces*, Ph. D. Thesis, Univ. of Arkansas, 1973.

[5] Crossley, S.G. and Hildebrand, S.K., *Semi-topological properties*, Fund. Math., **74** (1972), 233–254.

[6] Ganster, M., Noiri, T. and Reilly, I. L., *Weak and strong forms of θ-irresolute functions*, J. Inst. Math. Comput. Sci. Math. Ser., **1** (1988), 19–29.

[7] Garg, G.L. and Sivaraj, D., *Presemiclosed mappings*, Period. Math. Hungar., **19** (1988), 79–106.
[8] Khedr, F. H., *On semi-compact spaces*, Delta J. Sci., **8** (1984), 4–21.
[9] Levine, N., *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly, **70** (1963), 36–41.
[10] Njåstad, O., *On some classes of nearly open sets*, Pacific J. Math., **15** (1965), 961–970.
[11] Noiri, T., *A generalization of closed mappings*, Atti. Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., **54** (1973), 421–415.
[12] Takigawa, S., Ganster, M., Maki, H., Noiri, T. and Fujimoto, M. , *The digital n-space is quasi-submaximal*, Questions Answers Gen. Topology, **26** (2008), 45–52.
[13] Thompson, T., *S-closed spaces*, Proc. Amer. Math. Soc., **60** (1976), 335–338.

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