BOOK REVIEWS

A.N. Kolmogorov and A.P. Yushkevich, *Mathematics of the 19th Century. Mathematical Logic, Algebra, Number Theory, Probability Theory*, 2nd edition, Birkhäuser, 2001, Paperback, 308 pp., ISBN 3-7643-6442-4

Most histories of mathematics cover only schematic the period of the last two centuries. The reason is simple: there is to much material to be covered by a single author and even for two or three authors. In fact, the 19th century is the period when the branches of mathematics gained individuality. Also, at the end of the 18th century and at the beginning of the next one were active the last "encyclopedical" mathematicians (Euler, Gauss, Lagrange). Instead, for the first time, maybe, we may speak about "geometers" (such as Monge) or "algebraist" or "analysts".

As a consequences, we don't have anymore a single "history" of mathematics, but "histories" of separate branches, written by people involved, more or less, in the respective branches.

The book under review is the first volume (out of three) of such an approach to the history of mathematics. In spite of the name, it does not cover exactly the 19th century, but the period from the early 19th century up to the end of the 1930th. The reason is very good: 1801 and 1900 are not *turning points* in the history of mathematics, although some important books have been published in these two years.

As indicated by the title, this volume covers: logic, algebra, number theory and probability. More specifically, the first chapter is dedicated to mathematical logic (the Leibniz symbolic logic, the formal logic of de Morgan, the Boole's and Jevon's algebras of logic, the Venn's symbolic logic a.o.) and it is written by Z.A. Kuzicheva. The second chapter is concerned with algebra and algebraic number theory (the theory of algebraic equations, group theory, linear algebra, quaternions, algebras of Grassmann and Clifford, quadratic forms, algebraic numbers, ideals, beginning of commutative algebra). The authors are I.G. Bashmakova and A.N. Rudakov, with the assistance of A.N. Parshin and E.I. Slavutin. Chapter three is devoted to some problems in number theory (arithmetic theory of quadratic forms, geometry of numbers, analytical methods, transcedental numbers). The chapter is written by E.P. Ozhigova, with the assistance of A.P. Yushkevich. Finaly, chapter four is about probability theory and it is written by B.V. Gnedenko and O.B. Sheĭnin.

I should mention that this book (originally written in Russian), is the continuation of another multi-author work, devoted to the history of mathematics up to the beginning of the 19th century. The first editor of the book doesn't need, of course, any presentation. Let me add that the second editor, the late A.P. Yushkevich, was one of the best experts in the history of mathematics of the 20th century. Some of his works only were translated into other languages, but his name is well known to anyone interested in the history of mathematics and can read Russian.

This book (now at the second English edition), together with the other two volumes of this work, constitutes, in my opinion, the best description of the *entire* 19th century mathematics available in this moment.

A word of caution, though: in spite of the relatively big number of nice pictures scattered through the text, this is a history of mathematics for *mathematicians*, not just a collection of nice stories about mathematicians (in the line of Bell's *Men of mathematics*).

Paul A. Blaga

Erik M. Alfsen and Frederic W. Scultz, *State Spaces of Operator Algebras–Basic Theory, Orientations and C^*-products.* Mathematics: Theory and Applications, Birkhäuser Verlag, Boston-Basel-Berlin 2001, xii+350 pp., ISBN: 0-8176-3890-3 and 3-7643-3890-3.

The book is the first one (and the only) devoted to state spaces of C^*- and von Neumann algebras, its main purpose being to give geometric characterizations of these state spaces. The main notion used for these characterizations is that of global orientation of the minimal non-trivial faces of the state space. In the case of C^*-algebras these minimal faces are 3-dimensional Euclidean balls or line segments and one shows that there is a 1-1 correspondence between global orientations and C^*-products (meaning an associative product on a C^*-algebra giving the same state space). The minimal non-trivial faces of the state space together with orientability completely characterize the state space of C^*-algebras among JB-algebras (Jordan-Banach algebras). The case of von Neumann algebras is a little different, as the normal state space may have no extreme points. The result in this case was announced by the authors in Proc. Nat. Acad. Sci. USA in 1998, and appears here for the first time with complete proofs.

From their origins C^*- and von Neumann algebras were linked to quantum physics, the self-adjoint part of an algebra representing the observables. The decomposition of the associative product in the Jordan part and the Lie part reflects the dual role of physical variables–as observables and as generators of transformation groups. For this reason it is of great importance, both mathematically and physically, to study the connection between the Jordan part and the Lie part of a C^*-algebra in connection with state space orientations, which is one of the main themes of the book. In the Introduction the authors announces their intention to write another book, a sequel of the present one, giving a complete account of the geometric characterization of state spaces of C^*- and von Neumann algebras (and of their Jordan analogs, JB and JBW algebras).

The book is self-contained, the prerequisites being standard graduate courses in real and complex analysis, measure theory and functional analysis. The first two chapters, 1. *Introduction* and 2. *Elementary theory of C^*-algebras and von Neumann algebras*, can be viewed as a brief and fairly complete introduction to the theory of C^*- and von Neumann algebras. Beside an overview of some standard material from functional analysis, the first chapter contains also some more specialized results on order unit algebras and order derivations.

The rest of the chapters are devoted to state spaces and orientation: 3. *Ideals, faces and compressions*; 4. *The normal state space of $\mathcal{B}(H)$*; 5. *States, representations, and orientations of C^*-algebras*; 6. *Symmetries and rotations in von Neumann algebras*; 7. *Orientations and von Neumann algebras.*

Beside the basic material, included with complete proofs, each chapter ends with a section "Remarks", presenting topics which are of independent interest but are not needed later. Some of these are further mathematical results with sketched proofs or references to the literature, while the others discuss applications to physics.

Written by two specialists with fundamental contributions to the field, the book is addressed primarily to specialists in operator algebras, including those interested in applications to physics. By its introductory and self-contained character, the book can be used also by those wanting a quick introduction to C^*-algebras and von Neumann algebras.

S. Cobzaş

Frédéric Héléin, *Constant Mean Curvature Surfaces, Harmonic Maps and Integrable Systems.* Birkhäuser (Lectures in Mathematics, ETH Zürich), 2001, 122 pp., Softcover, ISBN 3-7643-6576-5

The most natural generalization of the notion minimal surface is that of a constant mean curvature surfaces. While they are not that simple as the surfaces of constant total curvature, the constant mean curvature surfaces play, nevertheless, an important role in the modern differential geometry. It became clear in recent years that constant mean curvature surfaces (in particular minimal surfaces) as well as harmonic maps are instances of integrable systems and many important properties of them can be obtained by taking this point of view.

The book under review, based on a series of lectures given by the author at Eidgenössisches Technische Hochschule from Zürich in the Spring of 1999, aims to give an introduction to the interplay between some classes of harmonic maps between a surface and a constant mean curvature surface or a symmetric manifold and the theory of completely integrable systems.

There is a closed connection between the harmonic maps and the constant mean curvature surfaces. For instance, a very nice result is that for a conformal immersion of a surface the Gauss map is harmonic if and only if the image of the immersion is a constant mean curvature surface.

The first six chapters of the book expose the basic material and tools from differential geometry, but also provide a brief survey of the connection between twistors and harmonic maps.

Probably the most important chapter of the book is the chapter seven, where the author investigates the harmonic maps as integrable systems. There is shown, in particular, that many of the properties of harmonic maps can be interpreted in terms of some suitable chosen loop groups operations.

In the next two chapters there are examined the so-called *finite type solutions*, in particular *finite type tori*.

The chapter ten discusses the so-called *Wente tori* (immersed constant mean curvature tori, constructed for the first time by Wente, in 1984), while the last chapter deals with a generalization of the Weierstrass type representation, initially constructed only for minimal surfaces.

The book is very clearly written and most of the important results (some of them belonging to the author) are proven. It provides a good introduction into the field (one of the first on this line of research) and should be recommended both to researchers in the field and graduate students (since the prerequisites are not very difficult to achieve).

Paul A. Blaga

Hyman Bass and Alexander Lubotzky, *Tree Lattices.* Progress in mathematics vol. 176, Birkhäuser Verlag, Basel-Boston-Berlin, 2001, xii+233 pp., Hardcover, ISBN 3-7643-4120-3.

Tree lattices, the main object of study in this book, are defined as follows. If X is a locally finite tree, then $G = \mathrm{Aut}(X)$ is a locally compact group, and the vertex stabilizers G_x are open and compact. A subgroup Γ of G is discrete if Γ_x is finite for every $x \in VX$, and in this case let

$$\mathrm{Vol}(\Gamma \backslash\backslash X) = \sum_{x \in \Gamma \backslash VX} \frac{1}{|\Gamma_x|}.$$

Then $|\Gamma|$ is called an X-*lattice* if $\mathrm{Vol}(\Gamma \backslash\backslash X) < \infty$, and in particular, a *uniform* X-lattice, if $\Gamma \setminus X$ is a finite graph. The technique which is used here is the theory of graphs of groups first introduced by Serre, and elaborated by the first author. Some of the most important examples for this theory arise from rank one simple Lie groups over a non-archimedean local field acting on their Bruhat trees, and there are also applications to combinatorics and number theory.

In this book, the authors present a systematic investigation of the existence, structure and properties of tree lattices, showing whenever possible the connections with the situation of lattices in Lie groups. Various examples are also constructed.

The book is essentially elementary and self-contained, and many results appear here for the first time in print. It will be a useful source of information and inspiration for graduate students interested in geometric methods in group theory, and especially to researchers in the field.

Andrei Marcus

Wolfgang Arendt, Charles J. K. Batty, Matthias Hieber, Frank Neubrander, *Vector-valued Laplace Transforms and Cauchy Problems.* Monographs in Mathematics, Birkhäuser Verlag, Boston-Basel-Berlin 2001, xi+523 pp., ISBN 3-7643-6549-8.

The aim of the present book is to apply the methods of vector-valued Laplace transforms to linear evolution equations in Banach spaces. In parallel, the theory of semigroups of operators and the related Cauchy linear problems are developed completely in the spirit of Laplace transforms. The book can be viewed as a sequel of the classical book of E. Hille and R. S. Phillips, *Functional Analysis and Semigroups,* Amer. Math. Soc., Providence 1957, but for functions with values in Banach spaces.

Consider the Cauchy equation

$$\text{(CP)} \qquad u'(t) = Au(t), \quad t \geq 0, \quad u(0) = x,$$

where A is a closed linear operator on a Banach space X and x is a given element of X. It turns out that u is a mild solution of (CP) if and only if

$$(\lambda - A)\hat{u}(\lambda) = x \quad (\lambda \quad \text{large}) \tag{1}$$

where $\hat{u}(\lambda) = \int_0^\infty \exp(-\lambda t)u(t)dt$ is the Laplace transform of u (supposed exponentially bounded). If λ is in the resolvent set of the operator A then

$$\hat{u}(\lambda) = (\lambda - A)^{-1}x. \tag{2}$$

This formula emphasizes the key role played by the spectral properties of the operator A for the Cauchy problem (CP), the link between solutions and resolvents, and between Cauchy problems and spectral properties of operators, being given by the Laplace transform.

The first part of the book, A. *Laplace transforms and Cauchy problems*, contains three chapters: 1. *The Laplace integral,* 2. *The Laplace transform,* 3. *Cauchy problems.* The first two chapters contain the basic results on Bochner, Riemann, and Riemann-Stieltjes integrals and Laplace transforms for functions defined on an interval $I \subset \mathbb{R}$ and with values in a Banach space X. Some geometrical properties of the space X (Radon-Nikodym property, containment of c_0) are discussed in connection with calculus rules and operational properties of the Laplace transform. The vector-valued Fourier transform on the line as well as the Cauchy problem for C_0-semigroups and for holomorphic semigroups are also considered. Again the proofs of two important results - the complex inversion formula for semigroups and Fattorini's theorem on square root reduction - require the restriction to an important class of Banach spaces, the UMD spaces, considered first by D. L. Burkholder in 1981 in a probabilistic framework (martingales in Banach spaces).

The second part of the book, B. *Tauberian theorems,* contains two chapters: 4. *Asymptotics of Laplace transforms,* and 5. *Asymptotics of solutions of Cauchy problems.* By Tauberian theorems one understands results relating the asymptotic behavior of $u(t)$ as $t \to \infty$ to that of $\hat{u}(\lambda)$ as $\lambda \downarrow 0$. Of particular interest are complex Tauberian theorems where the asymptotics involve the behavior of $\hat{u}(\lambda)$ for λ close to the imaginary axis. In Chapter 5, Tauberian theorems are applied to (CP) in various situations.

The third part of the book, C. *Applications and examples,* contains three chapters whose content is clear from their headings: 6. *The heat equations,* 7. *The wave equations,* 8. *Translation invariant operators on* $L^p(\mathbb{R}^n)$.

The prerequisites for the reading of the book are familiarity with basic results in functional analysis, theory of bounded linear operators, real and complex analysis. For the convenience of the reader four appendices are included. The first three collect together background material on vector-valued holomorphic functions, closed operators and ordered Banach spaces, while the fourth contains a proof of the Pelczyski's characterization of Banach spaces containing c_0.

Each chapter ends with a section of Notes, containing references to original works, historical remarks, presentations of related results and recommendations for further reading.

A notation list, an index and a large bibliography complete this treatise.

Written in a clear and accessible style and containing results obtained mainly in the last 15 years, the book is recommended to students and researchers in abstract evolution equations or interested in nontrivial applications of functional analysis.

S. Cobzaş

Y. André, and F. Baldassar, *De Rham Cohomology of Differential Modules on Algebraic Varieties.* Birkhäuser (Progress in Mathematics, 189), 2000, 214 pp., Hardcover, ISBN 3-7643-6348-7

The cohomology plays an important role in algebraic geometry. For a smooth complex algebraic variety there are three different variants of differential forms: algebraic, smooth and analytical and, correspondingly, there are three different de Rham cohomologies. The nice thing is that they are isomorphical. This has been generalized by Deligne to the case of cohomologies with non-constant coefficients, provided that the algebraic vector bundle of coefficients is endowed with an integrable connection which is, in some sense, regular. The book under review has as final aim the proof of a conjecture of Baldassari, regarding the more general setting of differential modules on algebraic varieties over a field of characteristic zero. The conjecture (theorem, in fact) claims that the algebraic and p-adic de Rham cohomologies for such a module are isomorphical.

To prove the theorem, an entire machinery had to be built and, in the process, the authors developed an algebraic theory of regularity and irregularity in several variables, give elementary proofs of some fundamental results regarding the de Rham cohomology of differential modules and other (more or less) classical results as the generalized Riemann existence theorem for coverings. Finally, they adapt the Artin's method of proving the comparison theorem for étale cohomology to the p-adic setting to proof the conjecture.

The book, a highly specialized monograph, is addressed to a rather narrow audience including experts and graduate students in arithmetic-algebraic geometry and D-modules. The first three chapters of the book (actually 3/4 of the whole) is of introductory nature and could serve as an introduction in the field for people having, nevertheless, a strong background in algebraic geometry.

Paul A. Blaga

Leonid Petrovich, *The Geometry of the Group of Symplectic Diffeomorphisms*. Lectures in Mathematics, Birkhäuser Verlag, Boston-Basel-Berlin, 2001, VII+134 pp, ISBN 3-7643-6432-7.

The book under review is concerned with the group of Hamiltonian diffeomorphisms of a symplectic manifold, which plays a fundamental role both in geometry and classical mechanics.

Under some assumptions on the manifold, this group is just the connected component of the identity in the group of all symplectic diffeomorphisms. ¿From the viewpoint of mechanics, this is the group of all admissible motions. What is the minimal amount of energy required in order to generate a given mechanical motion, i.e. a given Hamiltonian diffeomorphism? This variational problem admits an interpretation in terms of a remarkable geometry on the group discovered by Hofer in 1990. Hofer's geometry serves as a source of interesting problems and gives rise to new methods and notions which extend significantly our vision of the symplectic world. Since Hofer's work this new geometry has been intensively studied in the framework of symplectic topology with the use of modern techniques such as Gromov's theory of pseudo-holomorphic curves, Floer homology and Guillemin Sternberg-Lerman theory of symplectic connections. Furthermore, it opens up the intriguing prospect of using an alternative geometric intuition in dynamics.

In chapter 1 the author sums up some preliminary facts about the group of Hamiltonian diffeomorphisms. In chapter 2 biinvariant Finsler metrics on the group of Hamiltonian diffeomorphisms and Hofer's geometry are discussed. The purpose of chapter 3 and 4 is to prove – using Lagrangian submanifolds of symplectic manifolds – that Hofer's metric on $\Re^{2n}$ is non-degenerate. Chapter 5 is dedicated to the linearization of Hofer's geometry. In chapter 6 the theory of Lagrangian intersections is reviewed, which being combined with the linearization idea above give a rather powerful tool for investigation of the geometry of the group of Hamiltonian diffeomorphisms. In chapter 7 is proved that the group of Hamiltonian diffeomorphisms of a closed oriented surface has infinite diameter with respect to Hofer's metric. In the next chapter the asymptotic geometric behaviour of one-parameter subgroups of the Hamiltonian diffeomorphisms is discussed. Here is also presented a link between geometry and invariant tori of classical mechanics. In chapter 9 the author describes a method of calculation of the length spectrum in Hofer's geometry. In chapter 10 deformations of symplectic forms and pseudo-holomorphic curves are presented. In chapter 11 an asymptotic geometric invariant associated to the fundamental group $\mathrm{Ham}(M, \Omega)$ and an application to classical ergodic theory are presented. Chapter

12 contains some elements of variational theory of geodesics, and a visit to Floer homology is given in chapter 13. The last chapter presents the geometry of non-Hamiltonian diffeomorphisms.

This book is addressed to researchers and students from the graduate level onwards, and provides an essentially self-contained introduction into these developments and includes recent results on diameter, geodesics and growth of one-parameter subgroups in Hofer's geometry, as well as applications to dynamics and ergodic theory.

Ferenc Szenkovits

A. Dijksma, M.A. Kaashoek, A.C.M Ran Editors, *Revcent Advances in Operator Theory - The Israel Gohberg Anniversary Volume.* International Workshop on Operator Theory and Applications, IWOTA 98, lvii+558 pp., Operator Theory: Advances and Applications, Vol. 124, Birkhäuser Verlag, Boston-Basel-Berlin 2001, ISBN 3-7643-6573-0

The present volume is dedicated to the 70th birthday of Israel Gohberg and contains papers presented on this occasion at the International Workshop on Operator Theory (IWOTA-98) held in Groningen, the Netherlands, June 30 - July 3, 1998.

Israel Gohberg is one of the greatest personalities in operator theory–one of the initiators of IWOTA workshops and president for life of his steering committee, founder and editor of the international journal "Integral Equations and Operator Theory", founder and editor of the book series "Operator Theory: Advances and Applications" published by Birkhäuser Verlag, Basel, the author and co-author of 21 books and 401 articles. A list of his publications and a presentations of his main achievements, given by M.A. Kaashoek, are included. The volume contains also a CV of I. Gohberg and speeches delivered at the Conference Dinner.As a curiosity we mention the following fact from CV: in 1970 I. Gohberg was elected as a corresponding member of the Academy of Sciences of Moldova SSR, in 1974 he was removed from this academy, and reinstated in 1996 as a corresponding member of the Academy of Sciences of Moldova.

Beside this biographical material the volume contains also 25 contributed papers written by leading experts in the field all around the world and reflecting the wide range and the rich variety of topics presented and discussed at the workshop. These papers deal with operator polynomials and analytic operator functions, with spectral problems of (partial) differential operators and related operator matrices, with interpolation, completion and extension problems, with commutant lifting and dilation, with Ricatti equations and realization problems, with scattering theory, with problems from harmonic analysis, and with topics in the theory of reproducing kernel spaces and of spaces with an indefinite metric.

Containing results published for the first time and carefully reviewed before publication, the volume is of interest to a large audience of researchers in pure and applied mathematics, mainly in operator theory and applications.

S. Cobzaş

Franz Georg Timmesfeld, *Abstract Root Subgroups and Simple groups of Lie-Type.* Monographs in Mathematics vol. 95, Birkhäuser Verlag, Basel-Boston-Berlin, 2001, xiii+388 pp., Hardcover, ISBN 3-7643-6532-3.

In the classification theory of finite simple groups two methods emerged to be the most important. The first is the local group theoretic analysis created by J. Thompson, and the second is the "internal geometric analysis" created by B. Fischer and developed later by M. Aschbacher and the author of the present volume.

In this book he sistematically presents the theory of groups generated by a conjugacy class of subgroups satisfying certain generational properties on pairs of subgroups. Contrary to the local group theory, the geometric analysis was extended by the author to the case of infinite groups. The most important applications of the theory of abstract root subgroups are in the classification of simple classical and Lie-type groups, being related to the the theory of root subgroups of Chevalley groups.

The book is divided into five chapters. In Chapter I, rank one groups are studied. Such a group is generated by two different nilpotent subgroups A and B satisfying: for each $1 \neq a \in A$ there existes $b \in B$ satisfying $A^a = B^b$. These groups are the principal building blocks of groups of Lie Type. Chapter II introduces groups generated by abstract root subgroups and develops their theory. The main part of the book is Chapter III which consists of a comprehensive presentation of the classification of groups generated by abstract root subgroups. In Chapter IV a revision of the root involution classification is given. The last chapter discusses applications to quadratic pairs, to subgroups of classical and Lie type groups generated by long root elements, and to the determination of certain chamber transitive subgroups of Lie type.

The content of the of the book is largely due to the author, who is one of the main contributors to the topic. Complete proofs and new proofs of results that are unaccesible in the literature are given. This volume will be of a great value to students and researchers interested in finite group theory, classical groups, algebraic and Lie-type groups, buildings and generalized polygons.

Andrei Marcus