

LIPSCHITZ CONSTANTS OF SET-VALUED MAPPINGS
OBTAINED BY INTERSECTING CONVEX SETS WITH BALLS

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Abstract. We analyse the Lipschitz continuity of set-valued mappings defined on a Banach space X , obtained by intersecting convex sets with convenient balls depending on a parameter $\gamma > 1$. One obtains asymptotic estimates (with respect to γ) for Lipschitz constants when X is uniformly convex or uniformly smooth. General inequalities are expressed in terms of two moduli of Banach spaces. A characterization of Hilbert spaces involving Lipschitz constants for set-valued mappings is also given.

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