ON THE STABILITY OF FUNCTIONAL EQUATIONS ORIGINATED BY A PROBLEM OF ULAM

Themistocles M. Rassias

Abstract. During the last twenty years a number of results concerning the stability of various functional equations have been obtained. In particular, the stability problems of the additive functional equation and the Jensen's functional equation have been most intensively investigated. In this paper, we will introduce important results concerning the stability problems of the two functional equations.

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Key words. Stability, Hyers-Ulam stability, additive equation, Jensen's equation.

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National Technical University of Athens
Department of Mathematics
Zografou Campus
15780 Athens, Greece