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Geometric and analytic aspects of biholomorphic mappings in \mathbb{C}^n and complex Banach spaces

October 5, 2011–October 4, 2016

Director: Professor Gabriela Kohr

Faculty of Mathematics and Computer Science Babeş-Bolyai University, Cluj-Napoca Financed by National Authority for Scientific Research (ANCS), through CNCS-UEFISCDI

www.cncs-uefiscdi.ro; www.uefiscdi.ro Host institution of the project: Babeş-Bolyai University, Cluj-Napoca, Romania

A. Research team

- 1. Professor Gabriela Kohr (director)
- 2. Professor Mirela Kohr
- 3. Professor Paula Curt
- Dr. Teodora Chirilă (born Andrica) (Member as a PhD Student: October 5, 2011– January 31, 2014; member as a postdoc researcher: February 01, 2014–)

B. Project summary

The main focus of this project is the study of an important range in geometric function theory in finite and infinite dimensions related to biholomorphic mappings on the unit ball in \mathbb{C}^n and complex Banach spaces, by combining various methods in function theory in \mathbb{C}^n , semigroups theory, iteration theory, optimal control theory, univalent functions, etc. The thematic area of this project reflects the main research interest of its director in geometric function theory in \mathbb{C}^n and complex Banach spaces, and her research collaborations with well recognized mathematicians. The problems that we are going to investigate in the present research project are related to modern directions in geometric function theory in \mathbb{C}^n and complex Banach spaces.

C. Objectives

- I. Biholomorphic mappings on the unit ball in \mathbb{C}^n and complex Banach spaces
 - Objective 1. Abstract and modern applications of Loewner's theory in \mathbb{C}^n : Extension operators and L^d -Loewner chains. Applications to starlikeness and spirallikeness with respect to boundary points.

- Objective 2. New results and applications in the theory of univalent subordination chains, transition mappings, and the generalized Loewner differential equation in \mathbb{C}^n and complex Banach spaces.
- Objective 3. Applications of control theory and Loewner's theory in \mathbb{C}^n to study reachable families generated by the Carathéodory mappings.
- Objective 4. Construction of biholomorphic mappings on the unit balls in \mathbb{C}^n and complex Banach spaces, by applying various methods of function theory.
- Objective 5. The Carathéodory family of the unit balls in \mathbb{C}^n and complex Banach spaces. Analytic and geometric approaches.

II. Extension of classical results in the theory of biholomorphic mappings to the case of pluriharmonic mappings:

- Objective 6. Harmonic univalent mappings on the unit disc and pluriharmonic univalent mappings on the unit polydisc in \mathbb{C}^n . Necessary and sufficient conditions for global univalence.
- Objective7. Homeomorphic and quasiconformal extension of quasiregular pluriharmonic mappings on the unit ball in \mathbb{C}^n .

III. Applications of complex and harmonic analysis in fluid mechanics

• **Objective 8**. Modern applications of complex and harmonic analysis in the study of some special boundary value problems in fluid mechanics.

D. Contributions of the PhD student

Papers

- T. Chirilă, Subclasses of biholomorphic mappings associated with g-Loewner chains on the unit ball in Cⁿ, Complex Var. Elliptic Eqs., 59 (2014), 1456–1474.; Impact factor/2015: 0.466.
- 2. T. Chirilă, Analytic and geometric properties associated with some extension operators, Complex Var. Elliptic Eqs., 59 (2014), 427–442; Impact factor/2015: 0.466.
- T. Chirilă, An extension operator and Loewner chains on some Reinhardt domains in Cⁿ, Adv. Math.: Sci. Journal (AMSJ), 1 (2012), no.2, 139–145.
- T. Chirilă, H. Hamada, G. Kohr, Extreme points and support points for mappings with g-parametric representation in Cⁿ, Mathematica (Cluj), 56 (79), no. 1 (2014), 21-40.

PhD Thesis

 T. Andrica (căs. Chirilă), Contributions in the geometric function theory in Cⁿ, PhD theis, Babeş-Bolyai University, Cluj-Napoca (November 29, 2013). Adviser: Professor Gabriela Kohr.

Conferences

- 8th International Symposium on Geometric Function Theory and Applications, 27-31 August, 2012, Ohrid, Macedonia;
 T. Chirilă, Geometric properties associated with generalized Roper-Suffridge extension operators (communication).
- 9th International Symposium on Geometric Function Theory and Applications, August 26-30, 2013, Işik University, Istanbul, Turkey;
 T. Chirilă, Geometric properties of certain extension operators (communication).

E. Achievements

E1. ISI Publications

- F. Bracci, I. Graham, H. Hamada, G. Kohr, Variation of Loewner chains, extreme and support points in the class S⁰ in higher dimensions, Constructive Approx., 43 (2016), 231–251; Impact factor/2015: 1.346.
- I. Graham, H. Hamada, G. Kohr, M. Kohr, Support points and extreme points for mappings with A-parametric representation in Cⁿ, J. Geom. Anal., 26 (2016), 1560–1595; Impact factor/2015: 1.109.
- C.H. Chu, H. Hamada, T. Honda, G. Kohr, Distortion of locally biholomorphic Bloch mappings on bounded symmetric domains, J. Math. Anal. Appl., 441 (2016), 830–843; Impact factor/2015: 1.014.
- H. Hamada, M. Iancu, G. Kohr, Convergence results for families of univalent mappings on the unit ball in Cⁿ, Ann. Acad. Sci. Fenn. Math., 41 (2016), 601–616; Impact factor/2015: 0.83.
- H. Hamada, G. Kohr, Pluriharmonic mappings in Cⁿ and complex Banach spaces,
 J. Math. Anal. Appl., 426 (2015), 635–658; Impact factor/2015: 1.014.
- I. Graham, H. Hamada, G. Kohr, M. Kohr, Extremal properties associated with univalent subordination chains in Cⁿ, Math. Annalen, 359 (2014), 61–99; Impact factor/2015: 1.366.
- M. Chuaqui, H. Hamada, R. Hernández, G. Kohr, Pluriharmonic mappings and linearly connected domains in Cⁿ, Israel J. Math., 200 (2014), 489–506; Impact factor/2015: 0.738.
- I. Graham, H. Hamada, T. Honda, G. Kohr, K.H. Shon, Growth, distortion and coefficient bounds for Carathéodory families in Cⁿ and complex Banach spaces, J. Math. Anal. Appl., 416 (2014), 449–469; Impact factor/2015: 1.014.
- H. Hamada, G. Kohr, J.R. Muir, Extensions of L^d-Loewner chains in higher dimensions, J. Analyse Math., 120 (2013), 357–392; Impact factor/2015: 1.054.
- H. Hamada, T. Honda, G. Kohr, Growth and distortion theorems for linearly invariant families on homogeneous unit balls in Cⁿ, J. Math. Anal. Appl., 407 (2013), 398–412; Impact factor/2015: 1.014.

- I. Graham, H. Hamada, G. Kohr, M. Kohr, Asymptotically spirallike mappings in reflexive complex Banach spaces, Complex Analysis and Operator Theory, 7 (2013), 1909–1927; Impact factor/2015: 0.663.
- H. Hamada, G. Kohr, Univalence criterion and quasiconformal extension of holomorphic mappings, Manuscripta Math., 141 (2013), 195–209; Impact factor/2015: 0.591.
- H. Hamada, T. Honda, G. Kohr, Trace-order and a distortion theorem for linearly invariant families on the unit ball of a finite dimensional JB*-triple, J. Math. Anal. Appl., 396 (2012), 829–843; Impact factor/2015: 1.014.
- 14. P. Curt, Some remarks on certain invariant geometric properties in Hele-Shaw flows, Appl. Math. Comput., 236 (2014), 384–390; Impact factor/2015: 1.345.
- P. Curt, Janowski starlikeness in several complex variables and complex Hilbert spaces, Taiwanese J. Math., 18 (2014), 1171–1184; Impact factor/2015: 0.617.
- 16. P. Curt, On some invariant geometric properties in Hele-Shaw flows with small surface tension, Carpathian J. Math., **31** (2015), 53–60; Impact factor/2015: 0.61.
- P. Curt, D. Răducanu, General univalence criteria and quasiconformal extensions starting from Loewner chains theory, Filomat, 29 (2015), 1879–1892; Impact factor/2015: 0.603
- P. Curt, Invariant geometric properties in Hele-Shaw flows, Comput. Methods Funct. Theory, 16 (2016), 503–513; Impact factor/2015: 0.362.
- 19. T. Chirilă, Analytic and geometric properties associated with some extension operators, Complex Var. Elliptic Eqs., 59 (2014), 427–442; Impact factor/2015: 0.466.
- T. Chirilă, Subclasses of biholomorphic mappings associated with g-Loewner chains on the unit ball in Cⁿ, Complex Var. Elliptic Eqs., 59 (2014), 1456–1474; Impact factor/2015: 0.466.
- T. Chirilă, Extreme points, support points and g-Loewner chains associated with Roper-Suffridge and Pfaltzgraff-Suffridge extension operators, Complex Analysis and Operator Theory, 9 (2015), 1781–1799; Impact factor/2015: 0.663.

E2. Proceedings

 I. Graham, H. Hamada, G. Kohr, M. Kohr, Univalent subordination chains in reflexive complex Banach spaces, Contemporary Mathematics (AMS), 591 (2013), 83–111.

E3. Book chapters

2. I. Graham, H. Hamada, G. Kohr, Extremal problems for mappings with g-parametric representation on the unit polydisc in \mathbb{C}^n . In: New Trends and Open Problems in Complex Analysis and Dynamical Systems, Birkhäuser's series Trends in Mathematics, to appear.

E4. BDI publications

- T. Chirilă, H. Hamada, G. Kohr, Extreme points and support points for mappings with g-parametric representation in Cⁿ, Mathematica (Cluj), 56 (79), no. 1 (2014), 21-40.
- T. Chirilă, An extension operator and Loewner chains on some Reinhardt domains in Cⁿ, Adv. Math.: Sci. Journal (AMSJ), 1 (2012), no.2, 139–145.
- H. Hamada, T. Honda, G. Kohr, K.H. Shon, A note on strongly starlike mappings in several complex variables, Abstract Appl. Anal., Vol. 2014 (2014), Article ID 265718, 4 pp.; Impact factor/2013: 1.274.

E5. Preprints and papers submitted for publication

- 1. C.H. Chu, H. Hamada, T. Honda, G. Kohr, Bloch functions on bounded symmetric domains, submitted (2016).
- 2. H. Hamada, G. Kohr, α -Bloch mappings on bounded symmetric domains in \mathbb{C}^n , submitted (2016).
- 3. P. Curt, M. Kohr, Some geometrical properties of free boundaries in the Hele-Shaw flows, submitted (2016).
- I. Graham, H. Hamada, G. Kohr, M. Kohr, Bounded support points for mappings with g-parametric representation in C², submitted (2016).

E6. Findings

- Report-2011-romanian
- Report-2011-english
- Report-2012-romanian
- Report-2012-english
- Report-2013-romanian
- Report-2013-english
- Report-2014-romanian
- Report-2014-english
- Report-2015-romanian
- Report-2015-english
- Report-2016-romanian
- Report-2011-2016-english

E7. Research visits

- University of Toronto, Department of Mathematics; invited by Professor Ian Graham (April 23-May 14, 2012; August 16-August 29, 2012) (G. Kohr).
- University of Padova, Department of Mathematics, June 18-June 22, 2012; invited by Professor Massimo Lanza de Cristoforis (G. Kohr).
- University of Toronto, Department of Mathematics; invited by Professor Ian Graham (April 22-May 14, 2013; August 9-August 24, 2013) (G. Kohr).
- University of Toronto, Department of Mathematics; invited by Professor Ian Graham (April 22-May 13, 2014; August 5-August 22, 2014) (G. Kohr).
- Research visit to the University of Padova, Department of Mathematics, June 23-June 27, 2014, invited by Professor Massimo Lanza de Cristoforis (G. Kohr).
- University of Toronto, Department of Mathematics; invited by Professor Ian Graham (April 22-May 14, 2015; August 17-August 29, 2015) (G. Kohr).
- University of Stuttgart and University of Wuerzburg, May 28-May 31, 2015 (research visits), invited by Prof. W.L. Wendland and Prof. O. Roth (G. Kohr).
- University of Padova, Department of Mathematics, June 21-June 24, 2015, invited by Professor Massimo Lanza de Cristoforis (G. Kohr).
- University of Padova, Department of Mathematics, August 30-September 2, 2015, invited by Professor Massimo Lanza de Cristoforis (G. Kohr).
- University of Toronto, Department of Mathematics; invited by Professor Ian Graham (April 22-May 11, 2016; August 13-August 26, 2016) (G. Kohr).
- University of Padova, Department of Mathematics, July 25-August 4, 2016, invited by Professor Massimo Lanza de Cristoforis (G. Kohr).

E8. Conferences

- Harmonic and Complex Analysis and its Applications, Puerto de la Cruz, Tenerife, Canary Islands, Spain, March 5-9, 2012; http://hcaa2012.webs.ull.es/
 G. Kohr, Geometric and analytic aspects of Loewner chains in Cⁿ and complex Banach spaces (invited parallel session lecture).
- 9th International Conference on Harmonic Analysis and Partial Differential Equations, El Escorial, Madrid (Spain), June 11-15, 2012; http://www.uam.es/departamentos/ciencias/matematicas/AFA/Escorial2012/
 G. Kohr, The Loewner differential equation in several complex variables. Applications (poster; joint work with P. Curt and M. Kohr);
 P. Curt (participant).
- 8th International Symposium on Geometric Function Theory and Applications, 27-31 August, 2012, Ohrid, Macedonia; http://www.euro-math-soc.eu/node/2370
 T. Chirilă, Geometric properties associated with generalized Roper-Suffridge extension operators (communication).

• Tenth Advanced Course in Operator Theory and Complex Analysis, Sevilla, June 9-13, 2013;

http://congreso.us.es/ceacyto/2013/

G. Kohr, Extremal properties associated with the generalized Loewner differential equation in \mathbb{C}^n (contributed talk).

- Joint International Meeting of the AMS and the Romanian Mathematical Society, June 27- 30, 2013, Alba Iulia (Romania); http://imar.ro/ams-ro2013/description.php
 G. Kohr, Extremal properties associated with Loewner chains and the Loewner differential equation on the unit ball in Cⁿ (invited parallel session lecture).
- M. Kohr, Joint International Meeting of the AMS and the Romanian Mathematical Society, June 27-30, 2013, Alba Iulia (Romania).
- Joint International Meeting of the AMS and the Romanian Mathematical Society, June 27- 30, 2013, Alba Iulia (Romania);
 T. Chirilă, A subclass of biholomorphic mappings generated by g-Loewner chains (communication).
- 9th International ISAAC Congress, Krakow, August 5–9, 2013; http://www.isaac2013.up.krakow.pl/
 G. Kohr, Extremal properties associated with the generalized Loewner differential equation in Cⁿ (lecture).
- 9th International Symposium on Geometric Function Theory and Applications, Işik University, Istanbul (Turkey), August 26-30, 2013; http://gfta.isikun.edu.tr/
 G. Kohr, Loewner chains and extremal problems for bounded biholomorphic mappings with parametric representation in Cⁿ (invited speaker).
- 9th International Symposium on Geometric Function Theory and Applications, August 26-30, 2013, Işik University, Istanbul, Turkey; http://gfta.isikun.edu.tr/
 M. Kohr, Poisson problems for semilinear elliptic systems in Lipschitz domains. Applications (communication).
- 9th International Symposium on Geometric Function Theory and Applications, August 26-30, 2013, Işik University, Istanbul, Turkey; http://gfta.isikun.edu.tr/
 T. Chirilă, Geometric properties of certain extension operators (communication).
- 10th Joint Conference on Mathematics and Computer Science Cluj-Napoca, May 21-25, 2014; www.cs.ubbcluj.ro/ macs/2014/
 T. Chirilă, Extension operators that preserve certain geometric and analytic properties (communication).
- International Conference on Mathematics and Computer Science June 26-28, 2014, Braşov;

http://www.unitbv.ro/mi/CercetareStiintifica/Manifestari/MACOS.aspx **P. Curt**, *Invariant geometric properties in Hele-Shaw flows* (communication).

- Mini-courses in Mathematical Analysis 2012, Padova, June 23-27, 2014; http://minicourses.dmsa.unipd.it/
 G. Kohr, The generalized Loewner differential equation associated to univalent subordination chains in Cⁿ and complex Banach spaces. Applications (invited talk).
- 25th International Workshop of Operator Theory and its Applications (IWOTA 2014), Amsterdam, July, 14-18, 2014; http://www.math.vu.nl/ ran/iwota2014/
 G. Kohr, The generalized Loewner differential equation associated to univalent sub-ordination chains in Cⁿ and complex Banach spaces. Applications (contributed talk).
- Mini-courses in Mathematical Analysis 2015, Padova, June 22-26, 2015; http://minicourses.dmsa.unipd.it/
 G. Kohr, Extremal problems for univalent mappings with parametric representation in higher dimensions (invited talk).
- The Eighth Congress of Romanian Mathematicians, Iaşi, June 26-July 1, 2015; http://www.imar.ro/congmatro8/conf.php
 G. Kohr, The generalized Loewner differential equation in higher dimensions. Applications to extremal problems for biholomorphic mappings (contributed talk). Member in the scientific committee of the section Real and Complex Analysis, Potential Theory.
- International Conference on Nonlinear Operators, Differential Equations and Applications (ICNODEA), July 14-17, 2015, Cluj-Napoca, Romania; http://www.cs.ubbcluj.ro/ icnodeacj/

M. Kohr, Transmission problems for the Navier-Stokes and Darcy-Forchheimer-Brinkman systems in weighted Sobolev spaces on Lipschitz domains (Invited speaker).

T. Chirilă, Extreme, support points and g-Loewner chains associated with certain extension operators (poster session).

• International Conference on Complex Analysis and Related Topics. The 14th Romanian-Finnish Seminar, Bucharest, June 20-24, 2016; http://imar.ro/RoFinSem2016/conf.php

G. Kohr, Compact families of univalent mappings with parametric representation in \mathbb{C}^n (contributed talk); member of the organizing committee (G. Kohr).

M. Kohr, Boundary value problems for nonlinear Brinkman systems in Lipschitz domains (contributed talk).

• The 11th AIMS Conference on Dynamical Systems, Differential Equations and Applications, July 1-July 5, 2016, Orlando, Florida, USA; http://www.aimsciences.org/conferences/2016/

G. Kohr, Extremal problems, Loewner chains and the Loewner differential equation in \mathbb{C}^n and complex Banach spaces (**invited speaker** in the special session 94: "Infinite dimensional dynamics in analysis", organized by Cho-Ho Chu (London, UK)).

• The 14th International Conference on Integral Methods in Science and Engineering (IMSE 2016), 25–29 July, 2016, Padova; http://events.math.unipd.it/imse2016/

G. Kohr, *Loewner theory in the study of univalent mappings in higher dimensions* (conference talk).

- 12th International Symposium on Geometric Function Theory and Applications (GFTA 2016), August 25–28, 2016, Alba Iulia; http://gfta2016.uab.ro/index.php
 G. Kohr, Herglotz vector fields and univalent mappings on the unit ball in Cⁿ (invited speaker).
 G. Kohr, member in the scientific committee.
- XIII-ème Colloque Franco-Roumain de Mathématiques Appliquées, August 25–29, 2016, Iaşi (Romania); http://www.math.uaic.ro/cfr2016/index.php?info
 G. Kohr, member in the scientific committee.
- XIII-ème Colloque Franco-Roumain de Mathématiques Appliquées, August 25–29, 2016, Iaşi (Romania); http://www.math.uaic.ro/cfr2016/index.php?info
 M. Kohr, Boundary value problems for nonlinear Brinkman and Navier-Stokes equations with variable coefficients in Lipschitz domains (plenary speaker).
- International Conference on Mathematics and Computer Science (MACOS 2016), September 8–10 2016, Braşov (Romania); http://www.unitbv.ro/mi/CercetareStiintifica/Manifestari
 P. Curt, Invariant geometric properties of free boundaries in Hele-Shaw flows (contributed talk).
- INdAM Meeting "Geometric Function Theory in Higher Dimension", Cortona (Italy), 5–9 September, 2016; http://www.altamatematica.it/attivita
 G. Kohr, member in the scientific committee.

G. Kohr, Loewner theory in the study of univalent mappings in higher dimensions (invited talk).

E9. Seminar talks

- Seminar talk, University of Padova, Department of Mathematics, November 14, 2012;
 G. Kohr, Geometric and analytic approaches of Loewner chains in several complex variables. Applications. http://www.math.unipd.it/en/news/?id=1183/
- G. Kohr, The generalized Loewner differential equation. Applications, March 28, 2013; Seminar talk: Seminar of the Research Group on Mechanics and Astronomy, Babeş-Bolyai University, Cluj-Napoca. http://math.ubbcluj.ro/mecgrup/
- Seminar talk, University of Wuerzburg, Department of Mathematics, November 14, 2013;

G. Kohr, Loewner chains and extremal problems for biholomorphic mappings with parametric representation in \mathbb{C}^n . http://www.mathematik.uni-wuerzburg.de/ www4/research-seminar.html

• M. Kohr, Poisson problems for semilinear Brinkman systems on Lipschitz domains, June 6, 2013; Seminar talk: Seminar of the Research Group on Mechanics and Astronomy, Babeş-Bolyai University, Cluj-Napoca. http://math.ubbcluj.ro/ mecgrup/

- T. Chirilă, Univalent functions on the unit ball in \mathbb{C}^n , June 21, 2013; Seminar talk: Seminar of the Research Group on Function Theory, Babeş-Bolyai University, Cluj-Napoca.
- P. Curt, On some Hele-Shaw flow problems. Applications, November 6, 2014; Seminar talk: Seminar of the Research Group on Complex Analysis, Babeş-Bolyai University, Cluj-Napoca. http://www.math.ubbcluj.ro/ gctf/
- M. Kohr, Boundary value problems for nonlinear elliptic systems in domains with interior cuts. Applications, November 20, 2014; Seminar talk: Seminar of the Research Group on Mechanics and Astronomy, and Seminar of the Research Group on Complex Analysis, Babeş-Bolyai University, Cluj-Napoca. http://www.math.ubbcluj.ro/ mecgrup/; http://www.math.ubbcluj.ro/ gctf/
- G. Kohr, Support points and extreme points for mappings with A-parametric representation, December 4, 2014; Seminar talk: Seminar of the Research Group on Complex Analysis, Babeş-Bolyai University, Cluj-Napoca. http://www.math.ubbcluj.ro/gctf/
- M. Iancu, G. Kohr, Recent results in the theory of Loewner chains in higher dimensions, November 26, 2015; Seminar talk: Seminar of the Research Group on Complex Analysis, Babeş-Bolyai University, Cluj-Napoca. http://www.math.ubbcluj.ro/gctf/
- P. Curt, Invariant geometric properties in Hele-Shaw flows, December 3, 2015; Seminar talk: Seminar of the Research Group on Complex Analysis, Babeş-Bolyai University, Cluj-Napoca. http://www.math.ubbcluj.ro/gctf/
- M. Kohr, A layer potential approach for an inverse problem in fluid mechanics, December 3, 2015; Seminar talk: Seminar of the Research Group on Complex Analysis, Babeş-Bolyai University, Cluj-Napoca. http://www.math.ubbcluj.ro/ gctf/

External collaborators of Gabriela Kohr

- Ian Graham, Department of Mathematics, University of Toronto, Canada
- Hidetaka Hamada, Faculty of Engineering, Kyushu Sangyo University, Fukuoka, Japan
- Filippo Bracci, Dipartimento di Matematica, Università Di Roma "Tor Vergata", Rome, Italy

Prof. Dr. Filippo Bracci, Loewner theory and approximation of non-Runge mappings in higher dimensions, June 19, 2015; Seminar talk: Seminar of the Research Group on Complex Analysis, Babeş-Bolyai University, Cluj-Napoca. http://www.math.ubbcluj.ro/gctf/

Contact

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