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# YUKAWA: THE MAN AND THE POTENTIAL

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**Abstract.** We expose the biography of the Nobel laureate from Japan, Hideki Yukawa, who predicted the existence of pi-mesons in his first scientific paper, written at the age of 28. Afterwards, he met the most prominent physicists of the time, but he came back to Japan and promoted new areas of science. He was aware of scientists' responsibility for preventing the proliferation of nuclear weapons. The potential that bears his name originates in the mentioned paper, and it is important in Nuclear Physics and Astrophysics. Due to its simple form, it can be studied by students, even in high school.

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### 1. INTRODUCTION

Theorems, spaces, various objects in Mathematics, Physics and other domains bear the names of people who lived or are still living. It is important for professors to spend some of their time to look at those persons biographies, both scientific and personal. They will be rewarded instantly by the reading itself, and afterwards by enlarging their understanding of our world and also by teaching the students something beyond the strict curriculum.

Yukawa appears as a potential's name in Nuclear Physics and Astrophysics. Yukawa the man was the first Nobel Prize medalist in Japan, a devoted professor of Theoretical Physics and a person who was concerned about maintaining world's peace.

## 2. BIOGRAPHY OF A NOBEL PRIZE MEDALIST

Many papers and sites are dedicated to the life and work of the scientist Yukawa, such as [1], [3], [4], [6] or [7].

Hideki Yukawa was born on January 23, 1907 at Tokyo, in Takuji and Koyuki Ogawa family. Both parents came from scholar families of the samurai tradition. Hideki was the fifth of seven children (five boys and two girls). The youngest brother was killed as a soldier in the WWII, but all the other four boys became scholars. Three of them had been professors at Kyoto University simultaneously. Ogawa family moved to Kyoto in 1908 because Hideki's father, Takuji Ogawa, was recruited to teach geography at the Faculty of Literature



in Kyoto University, founded in 1906. Later on he moved to the Faculty of Science as a geologist.

Hideki was raised and attended school in Kyoto. He read some books on modern physics while in high school and he was attracted by this domain. He entered Kyoto Imperial University (now Kyoto University) in 1926, and, showing his intelligence early, graduated only three years later with a master's degree. Since that time he has been engaged in theoretical physics investigations, particularly in the theory of elementary particles. He spent the next ten years continuing his education and teaching. After three years of research, he got in 1932 an appointment as a physics lecturer at Kyoto University. Next, he became assistant professor at Osaka University, where he received the Ph. D. in Physics in 1938. From the following year he was a Professor of Theoretical Physics at Kyoto University.

In 1932 he married Sumi Yukawa and he changed his surname, taking that of his wife family. They had two sons, Harumi and Takaaki.

While at Osaka University, at the age of 28, he published in 1935 his first paper [5], in which he proposed a new field theory of nuclear forces and predicted the existence of the meson. Encouraged by the discovery by American physicists of one type of meson in cosmic rays, in 1937, he devoted himself to the development of the meson theory, on the basis of his original idea. But the details of his theory did not correspond with the measured properties of that particle. It was not until 1947 that the pion was discovered. Finally, Yukawa's theory of nuclear force was validated, and in 1949 he received the Nobel Prize in Physics "for his prediction of the existence of mesons on the basis of theoretical work on nuclear forces". Yukawa discussed his contributions to meson theory in his Nobel lecture, reprinted in Nobel Lectures in Physics, vol. 3 (1964). He was the first Japanese to receive the Nobel Prize. In 1939, Yukawa was invited to the eighth Solvay Conference and he began his first travel to a foreign country. This series of conferences, starting from 1911, is named after Ernest Solvay (April 16, 1838 – May 26, 1922), a Belgian chemist, industrialist and philanthropist, and continues till our days (the 26th, on Astrophysics and Cosmology, was held in 2014). Unfortunately on September 1st, the world war in Europe had outbroken by the invasion of Hitler's troops into Poland, and Solvay Conference and all the other invitations to Yukawa in Europe were cancelled. The eighth Solvay Conference was held not earlier than 1948. All the Japanese who happened to be in Central Europe had been evacuated into USA. Yukawa landed at New York and crossed the continent until San Francisco. Between September 14 - October 13, he visited nine universities and met many eminent physicists, like Fermi, Szilard, Wigner, Wheeler, Gamow, Teller, Oppenheimer, etc.

Two years after Yukawa's round-the-world trip, Japan-USA war outbroke in December, 1941 and the academic activity slowed down. The war ended by the strikes of the Atomic Bomb to Hiroshima and Nagasaki. Furthermore, the occupation USA army prohibited all researches on nuclear physics and the three cyclotrons existing at the time were destroyed and sunk into the sea. Yukawa was invited by Oppenheimer in 1948 as Visiting Professor to the Institute for Advanced Study at Princeton, USA. There Yukawa met Albert Einstein, with whom he remained friends for the rest of the older scientist's life. Since July 1949 he has been Visiting Professor at Columbia University, New York City. In 1949, Yukawa was awarded the Nobel Prize, producing a positive impact to all Japanese, who had been depressed by the surrender. He suddenly became a national hero.

In 1953 he returned home to Kyoto to become director of a new research institute. At the inauguration of the institute, many leading physicists attended the International Conference on Theoretical Physics. As the director of the institute, Yukawa also promoted new areas of science, such as astrophysics, space science or biophysics. In 1955, the first workshop on astronomy and nuclear-particle physics was held under Yukawa's initiative. Then the research on nuclear astrophysics had developed rapidly in Japan, both theoretically and experimentally.

Yukawa had strong feelings on science's duty to humanity. In 1949, when he won the Nobel Prize, he chose to donate most of his award money to several institutions in Japan, including the Research Institute for Fundamental Physics at Kyoto University. He was one of the eleven signatories of the *Russell– Einstein Manifesto* from 1955, meant to call for a conference where scientists would assess the dangers posed to the survival of humanity by weapons of mass destruction. This was the starting point for Pugwash Conferences on Science and World Affairs, the first being held in July 1957. Yukawa believed that at least some of the responsibility for preventing war must rely on the scientists who produce its technology. He expressed this in 1962, at the Kyoto

Conference of Scientists: The results of physics are inevitably connected with the problems of humanity through their application to human society.

He published a large number of scientific papers and many books, including *Introduction to Quantum Mechanics* (1946) and *Introduction to the Theory of Elementary Particles* (1948), both in Japanese. Since 1946, he has edited a journal in English, Progress of Theoretical Physics, which had contributed to spread the research results of Japan in the scientific world.

Beside the textbooks and scientific papers, he wrote essays on culture and was very much concerned with world peace problem. He had conversations with prominent scholars and artists of his time, which were recorded and published, and sometimes broadcasted on radio and TV. Yukawa made also Tanka (a sort of Japanese poems) and was a master of calligraphy ([3]).

He retired from his position in 1970 and died in Kyoto on September 8, 1981.

The academic societies of his country have recognized his outstanding merits and he was elected as member of the Japan Academy, of the Physical Society and of the Science Council of Japan. He was Emeritus Professor of Osaka University. As Director of the Research Institute for Fundamental Physics in Kyoto University he had his office in the Yukawa Hall, which was named after him.

The Imperial Prize of the Japan Academy was awarded to Yukawa in 1940; he received the Decoration of Cultural Merit in 1943, and the Japanese government's Order of the Rising Sun in 1977. A civic honour was also awarded to him, namely Honorary Citizen of the City of Kyoto.

His work was recognized all over the world. He was a foreign member of the American National Academy of Sciences and Fellow of the American Physical Society. An honorary doctorate of the University of Paris and honorary memberships of the Royal Society of Edinburgh, the Indian Academy of Sciences, the Russian Academy of Sciences, the International Academy of Philosophy and Sciences, and the Pontificia Academia Scientiarum have marked the recognition he has earned in world scientific circles. He won the Lomonosov Gold Medal of the Russian Academy of Sciences and the Order of Merit of the Federal Republic of Germany.

UNESCO was associated in 2007 with the 100th anniversary of Yukawa's birth ([8]). A medal was struck by the Paris Mint, the obverse featuring a portrait of the late physicist, while the reverse having the UNESCO logo. It was designed by renowned Japanese painter Ikuo Hirayama.

A lot of stamps were dedicated to Yukawa all over the world.

It is interesting to mention how the life and career of Yukawa intertwined with that of Sin-itiro Tomonaga (March 31, 1906, Kyoto – July 8, 1979, Tokyo) [4]. Their fathers were both recruited to teach at the Faculty of Literature in Kyoto University (Tomonaga's father lectured western philosophy). They were classmates in high school, and in 1926 they were enrolled and studied together at the Department of Physics of Kyoto University. In 1939 they

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met in Germany, where Tomonaga was studying in Heisenberg's laboratory in Leipzig, and then travelled together to the USA.

After the WWII, Yukawa and Tomonaga took their responsibility to rebuilt the academic system on the destructed campus. At the year of the surrender of Japan, they were yet before 40 years old.

They delivered lectures for students, particularly in Quantum Mechanics, and wrote many textbooks. Both Yukawa and Tomonaga authored a lot of articles and essays not only in science, but also on culture, education and peace problems. The collections of their writings amount more than ten volumes each. Yukawa and Tomonaga acted tirelessly for nuclear disarmament movement, such as *Pugwash Movement*, being two of the twenty-two scientists who attended the first conference held in July 1957 in Pugwash, Nova Scotia. And they were both Nobel laureates, Yukawa in 1949 and Tomonaga in 1965, as the second Japanese recipient. The 1965 Nobel Prize in Physics was awarded to Sin-Itiro Tomonaga, Julian Schwinger and Richard P. Feynman "for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles".

#### 3. YUKAWA POTENTIAL

Yukawa was interested in the forces inside the atoms nuclei. In his article [5], Yukawa argued that the nuclear strong force is carried by an unknown yet particle with a mass approximately 200 times that of an electron. Shortly after Yukawa's prediction, a particle with almost precisely that mass was discovered in 1937 in cosmic ray phenomena. Although its mass was 207 times that of an electron, it was not the particle predicted by Yukawa, but a heavy electron, which is now called muon. Later on, in 1947, three particles with masses approximately 270 times that of an electron were found in cosmic rays. These did have the properties that Yukawa had predicted. One was of positive charge, one of negative charge and one was neutral. They were called pi-mesons but now they are known as *pions*.

The discovery of pions was due to mainly to three people. Cesare Lattes (July 11, 1924 – March 8, 2005), being in 1947 at a weather station on top of the 5,200-meter high Chacaltaya mountain in Bolivia, used photographic plates to register the cosmic rays. Lattes and his Italian professor Giuseppe Occhialini (December 5, 1907 – December 30, 1993) brought the plates to the H. H. Wills Laboratory of the University of Bristol, directed by Cecil Powell (December 5, 1903 – August 9, 1969). There, they made the great experimental discovery, using Lattes' plates and Powell's improved photographic emulsions: the pion (or pi meson). In the same year, Lattes calculated the new particle's mass. In 1948, working with Eugene Gardner at University of California, Berkeley, Lattes was able to detect the artificial production of pions in the cyclotron, by bombarding carbon atoms with alpha particles. He was just 24 years old. In 1950, Cecil Powell was awarded the Nobel Prize in

Physics "for his development of the photographic method of studying nuclear processes and his discoveries regarding mesons made with this method".

Yukawa noticed that the interaction between the elementary particles in the nucleus can be described by means of a force field, just as the interaction between the charged particles is described by the electromagnetic field. In quantum theory, this field should be accompanied by a new sort of quantum, just as the electromagnetic field is accompanied by the photon - but this quantum must be very massive. The potential of force between the neutron and the proton should not be of Coulomb type, but decreasing much more rapidly with distance. The constant  $\lambda$ , having the dimensions of an inverse length, determines how the potential falls off with the distance r. The reciprocal of  $\lambda$ is identified with the range of the potential, for the potential is practically null at distances greater than  $1/\lambda$ . Assuming  $\lambda = 5 \times 10^{12}$  cm<sup>-1</sup>, Yukawa obtained for the mass of the new quantum a value  $2 \times 10^2$  times as large as the electron mass. The pion, found in cosmic rays twelve years later by Lattes, Occhialini and Powell, has the adequate mass.

The Coulomb force has, up to a sign, the potential energy

(1) 
$$V(r) = \frac{1}{r}, r > 0.$$

Yukawa proposed a potential, which we denote by  $Y_{\lambda}$ , of the form

(2) 
$$Y_{\lambda}(r) = \frac{\exp(-\lambda r)}{r}$$

where  $\lambda$  is a positive parameter subject to physical measurement.

Yukawa notes that the potential V is a static centrally symmetric solution of the wave equation

(3) 
$$\left(\nabla^2 - \left(\frac{1}{c^2}\right)\frac{\partial^2}{\partial t^2}\right)U = 0,$$

where  $\nabla^2$  is the Laplacian operator,

(4) 
$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

and c the light velocity. His potential, in turn, is a static centrally symmetric solution of the equation

(5) 
$$\left(\nabla^2 - \left(\frac{1}{c^2}\right)\frac{\partial^2}{\partial t^2} - \lambda^2\right)U = 0.$$

In order to draw the graph of  $Y_{\lambda}$  for a fixed  $\lambda$ , we calculate  $\lim_{r\to 0+} Y_{\lambda}(r) = +\infty$  and  $\lim_{r\to+\infty} Y_{\lambda}(r) = 0$  and we prepare the derivatives

(6) 
$$Y'_{\lambda}(r) = -\frac{(\lambda r+1)\exp(-\lambda r)}{r^2}, \quad Y''_{\lambda}(r) = \frac{((\lambda r+1)^2+1)\exp(-\lambda r)}{r^3}.$$



FIG. 3.1 – Graphs of V (black),  $Y_1$  (blue),  $Y_2$ (red),  $Y_{10}$ (green)

The function  $Y_{\lambda}$  is convex, decreasing from  $+\infty$  at 0 to 0 at  $+\infty$ . Its graph has a similar shape to that of V. Because for r > 0 it is  $\exp(-\lambda r) < 1$ ,

(7) 
$$Y_{\lambda}(r) < V(r), \quad \forall r > 0$$

holds whatever  $\lambda > 0$ . We have also, for  $\lambda_1 < \lambda_2$ ,

(8) 
$$Y_{\lambda_1}(r) > Y_{\lambda_2}(r), \quad \forall r > 0.$$

As an exercise, the same task may be proposed to students for  $\lambda < 0$ .

In Figure 1 one can see that Yukawa potential decays strongly and it acts only in the vicinity of the origin (the larger  $\lambda$  is, the smaller is r for which the action of the potentials is significant).

We mention that a potential of the form (2) was proposed by van der Waals (a Nobel Prize recipient) in 1893 to represent intermolecular forces. Max Born, a Nobel Prize laureate too, writes in his autobiography that he pondered on the potential (2) in his student's days when he found it in Minkowski's Encyclopaedia in the article on capillarity, but he missed the possible particle interpretation.

In Astrophysics, potentials of the form (2) are used as corrections for the Newtonian potential to explain recently observed phenomena that occur at various scales, from solar system to cosmological distances, which have not yet found an explanation in terms of conventional physics. For example, a potential of the form

(9) 
$$W_{\lambda}(r) = \frac{1}{r} + a \frac{\exp(-\lambda r)}{r},$$

where  $\lambda$ , *a* are positive parameters, is considered in [2].

In order to draw the graph of  $W_{\lambda}$ , we calculate  $\lim_{r\to 0+} W_{\lambda}(r) = +\infty$  and  $\lim_{r\to +\infty} W_{\lambda}(r) = 0$  and we prepare the derivatives

(10) 
$$W'_{\lambda}(r) = -\frac{1+a(\lambda r+1)\exp(-\lambda r)}{r^2}, \ W''_{\lambda}(r) = \frac{2+a\left((\lambda r+1)^2+1\right)\exp(-\lambda r)}{r^3}$$

The function is convex, decreasing from  $+\infty$  at 0 to 0 at  $+\infty$ . Its graph has a similar shape to that of V. Because of a > 0,

(11)  $V(r) < W_{\lambda}(r), \quad \forall r > 0$ 

holds whatever  $\lambda > 0$ . We have also, for fixed a > 0 and  $\lambda_1 < \lambda_2$ ,

(12) 
$$W_{\lambda_1}(r) > W_{\lambda_2}(r), \quad \forall r > 0.$$

These properties are illustrated in Figure 2.



Figure 2: Graphs of V (black),  $W_1$  (blue),  $W_2$ (red),  $W_3$ (green)

The graphs in Figure 1 and 2 can be drawn by high-school students, and more facts on their meaning can be explained to university students. Some words on the life of Yukawa will be in the benefit of all of them.

#### REFERENCES

- [1] Brown, L. M., Yukawa's prediction of the meson, Centaurus 25 (1981), 71-132
- [2] Haranas, I., Ragos, O., Mioc, V., Yukawa-type potential effects in the anomalistic period of celestial bodies, Astrophysics and Space Science 332 (1) (2011), 107-113
- [3] Nambu, I., The legacies of Yukawa and his disciples, Nuclear Physics A 805 (2008), 90c-97c
- [4] Sato, H., Biography of Hideki Yukawa, Nuclear Physics A 805 (2008), 21c-28c
- [5] Yukawa, H., On the interaction of elementary particles, Proc. Phys. Math. Soc. Japan 17 (1935), 48-57
- [6] http://biography.yourdictionary.com/hideki-yukawa
- [7] http://www.nobelprize.org/nobel\_prizes/physics/laureates/1949/yukawa-bio.html
- [8] http://portal.unesco.org/en/ev.php-URL\_ID=26478&URL\_DO= DO\_TOPIC&URL\_SECTION=201.html

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