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APPROXIMATION METHODS FOR TRIPLE HIERARCHICAL VARIATIONAL INEQUALITIES (II)

L.-C. CENG*, Q.H. ANSARI**, A. PETRUŞEL*** AND J.-C. YAO****

*Department of Mathematics, Shanghai Normal University, Shanghai 200234 and

Scientific Computing Key Laboratory of Shanghai Universities, China E-mail: zenglc@hotmail.com

**Department of Mathematics, Aligarh Muslim University, Aligarh 202 002, India and

Department of Mathematics and Statistics, King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia E-mail: qhansari@gmail.com

> ***Department of Mathematics, Babeş-Bolyai University Cluj-Napoca Kogălniceanu Street, No.1, 400084 Cluj-Napoca, Romania E-mail: petrusel@math.ubbcluj.ro

**** (Corresponding author) Center for General Education, China Medical University Taichung 40402, Taiwan and

> Department of Mathematics, King Abdulaziz University P.O. Box 80203, Jeddah 21589, Saudi Arabia E-mail: yaojc@mail.cmu.edu.tw

Abstract. In this paper, we consider a triple hierarchical variational inequalities (in short, THVI) with a finite family of nonexpansive mappings. By combining the viscosity approximation method, hybrid steepest-descent method and Mann's iteration method, we propose the hybrid steepest-descent viscosity approximation method for solving the THVI. The strong convergence of this method to a unique solution of the THVI is studied. Under some mild conditions, a strong convergence result (to the unique solution of THVI) for another iterative algorithm is also presented.

Key Words and Phrases: Triple hierarchical variational inequalities, hybrid steepest-descent viscosity approximation method, monotone operators, nonexpansive mappings, fixed point, strong convergence theorem.

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