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APPROXIMATION METHODS FOR TRIPLE HIERARCHICAL VARIATIONAL INEQUALITIES (I)

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Abstract. In this work, we consider two types of triple hierarchical variational inequalities (in short, THVI), one with a single nonexpansive mapping and another one with a finite family of nonexpansive mappings. In this paper, by combining the viscosity approximation method, hybrid steepest-descent method and Mann's iteration method, we propose the hybrid steepest-descent viscosity approximation method for solving the THVI. The strong convergence of this method to a unique solution of the THVI is studied under some appropriate assumptions. Another iterative algorithm for solving THVI is also presented. Under some mild conditions, we prove that the sequence generated by the proposed algorithm converges strongly to a unique solution of THVI. The case of a finite family of nonexpansive mappings will ve presented in the second part of this work.

Key Words and Phrases: Triple hierarchical variational inequalities, hybrid steepest-descent viscosity approximation method, monotone operators, nonexpansive mappings, fixed points, strong convergence theorems.
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