Book Review

W.A. Kirk and N. Shahzad: Fixed Point Theory in Distance Spaces, Springer, Heidelberg New-York Dordrecht London, 2014, xi + 173 pp, ISBN 978-3-319-10926-8; ISBN 978-3-319-10927-5 (eBook); DOI 10.1007/978-3-319-10927-5.

The book is devoted to various aspects of fixed point theory in metric spaces and other more general structures. The structure of the book is the following: three main parts: I. *Metric Spaces*, II. *Length Spaces and Geodesic Spaces*, and III. *Beyond Metric Spaces*, preceded by a nice Preface and followed by a consistent References list with 223 titles.

The main topics of the first part are Caristi's fixed point theorem and some properties of nonexpansive operators. A special attention is paid to the question whether a proof depends on the axiom of choice or on some its weaker forms (Axion of Dependent Choice and Axion of Countable Choice), a theme which appears recurrently throughout the book. The fixed point theory for nonexpansive mappings is discussed within the context of metric spaces endowed with a compact and normal convexity structure. The proof is based on Zermelo's fixed point theorem in ordered sets, requiring only Zermelo-Fraenkel set theory and the Axiom of Dependent Choice. The first part ends with fixed point results for nonexpansive mappings on hyperconvex metric spaces, with emphasis on hyperconvex ultrametric spaces (metric spaces (X, d) with $d(x, y) \leq \max\{d(x, z), d(z, y)\}$ for all $x, y, z \in X$).

The second part is concerned with spaces which, in addition to their metric structure, also have a geometric structure, such as length spaces, geodesic spaces, Busemann spaces, CAT(k) spaces, Ptolemaic spaces and $\mathbb{R}$-trees.

The focus of the third part is on the study of fixed points for operators on b-metric spaces (also called quasimetric spaces), generalized metric spaces (in the sense of Branciari) and partial metric spaces. Recall that, a semimetric is a function $d : X \times X \to \mathbb{R}_+$ such that (i) $d(x, y) = 0 \iff x = y$ and (ii) $d(x, y) = d(y, x)$, for all $x, y \in X$. A b-metric space with constant $s \geq 1$ is a semimetric space (X, d) such that $d(x, y) \leq s[d(x, y) + d(z, y)]$ for all $x, y \in X$. Various aspects of fixed point theory for singlevalued and multivalued operators in these generalized metric spaces are presented.

The book is interesting, clearly written and contains many important results (most of them of the authors of this book) in the field of applied nonlinear analysis. Of course, the focus is on fixed point theory in (generalized) metric spaces. The sources of the presented results are carefully mentioned and interesting open questions are pointed out for further investigations. The book will be an important reference tool for researchers working in fixed point theory and related topics, as well as, for those interested in applications of this theory in other areas, such as computer science, biology, economics.

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