

THE n -DIMENSIONAL U -CONVEXITY AND GEOMETRY OF BANACH SPACES

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Abstract. In this paper, we introduce the modulus of n -dimensional U -convexity which simultaneously generalizes modulus of n -dimensional uniform convexity due to Kirk [14] and modulus of U -convexity due to Gao [9]. The properties of the modulus are investigated and the relationships between this modulus and other geometric properties of Banach spaces are studied. Some results on fixed point theory for nonexpansive mappings and normal structure in Banach spaces are improved.
Key Words and Phrases: Fixed point property, modulus of n -dimensional uniform convexity, modulus of n -dimensional U -convexity, modulus of U -convexity, nonexpansive mapping, normal structure.

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REFERENCES

- [1] J.S. Bae, M.S. Park, *On the k -characteristic of convexity*, Analysis and Geometry 1989 (Taejŏn, 1989), 159–165, Korea Inst. Tech., Taejŏn, 1989.
- [2] J. Bernal, F. Sullivan, *Multidimensional volumes, super-reflexivity and normal structure in Banach spaces*, Illinois J. Math., **27**(1983), no. 3, 501–513.
- [3] B. Bollobás, *An extension to the theorem of Bishop and Phelps*, Bull. London Math. Soc., **2**(1970), 181–182.
- [4] J.A. Clarkson, *Uniformly convex spaces*, Trans. Amer. Math. Soc., **40**(1936), no. 3, 396–414.
- [5] D. Dacunha-Castelle, J.L. Krivine, *Applications des ultraproducts à l'étude des espaces et des algèbres de Banach*, (French) Studia Math., **41**(1972), 315–334.
- [6] M.M. Day, *Normed Linear Spaces*, Third edition, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 21, Springer-Verlag, New York-Heidelberg, 1973.

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- [7] J. Diestel, *Geometry of Banach Spaces, Selected Topics*, Lecture Notes in Mathematics, Vol. 485, Springer-Verlag, Berlin-New York, 1975.
- [8] J. García-Falset, *The fixed point property in Banach spaces with the NUS-property*, J. Math. Anal. Appl., **215**(1997), no. 2, 532–542.
- [9] J. Gao, *Normal structure and modulus of U -convexity in Banach spaces*, Function spaces, differential operators and nonlinear analysis (Paseky nad Jizerou, 1995), 195–199, Prometheus, Prague, 1996.
- [10] R. Geremia, F. Sullivan, *Multidimensional volumes and moduli of convexity in Banach spaces*, Ann. Mat. Pura Appl., **127**(1981), no. 4, 231–251.
- [11] K. Goebel, *Convexity of balls and fixed-point theorems for mappings with nonexpansive square*, Compositio Math., **22**(1970), 269–274.
- [12] R.C. James, *Weakly compact sets*, Trans. Amer. Math. Soc., **113**(1964), 129–140.
- [13] M.A. Khamsi, *Uniform smoothness implies super-normal structure property*, Nonlinear Anal., **19**(1992), no. 11, 1063–1069.
- [14] W.A. Kirk, *The modulus of k -rotundity*, Boll. Un. Mat. Ital. A, **7**(1988), no. 2, 195–201.
- [15] T.-C. Lim, *On moduli of k -convexity*, Abstr. Appl. Anal., **4**(1999), no. 4, 243–247.
- [16] E.V. Mazcuñán-Navarro, *On the modulus of u -convexity of Ji Gao*, Abstr. Appl. Anal., **2003**(2003), no. 1, 49–54.
- [17] E.M. Mazcuñán-Navarro, *Geometry of Banach Spaces in Metric Fixed Point Theory*, Ph.D. Thesis, 2003, 41–57.
- [18] S. Saejung, *On the modulus of U -convexity*, Abstr. Appl. Anal., **2005**(2005), no. 1, 59–66.
- [19] S. Saejung, *Convexity conditions and normal structure of Banach spaces*, J. Math. Anal. Appl., **344**(2008), no. 2, 851–856.
- [20] E. Silverman, *Definitions of Lebesgue area for surfaces in metric spaces*, Rivista Mat. Univ. Parma, **2**(1951), 47–76.
- [21] B. Sims, *"Ultra"-techniques in Banach space theory*, Queen's Papers in Pure and Applied Mathematics, 60, Queen's University, Kingston, ON, 1982.
- [22] F. Sullivan, *A generalization of uniformly rotund Banach spaces*, Canad. J. Math., **31**(1979), No.3, 628–636.

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