

NOTE ON MULTIFUNCTIONS CONDENSING IN THE HYPERSPACE

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Dedicated to the memory of Professor Francesco S. De Blasi

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Abstract. We fill the gap in understanding the relationship between mappings which are condensing w.r.t. the measure of noncompactness defined on the hyperspace and multifunctions condensing in the ordinary sense.

Key Words and Phrases: Hyperspace, measure of noncompactness, condensing multifunction, hyper-condensing multifunction, strongly condensing multifunction, strongly hyper-condensing multifunction, Nadler contraction, h-compact multifunction.

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