

## ON THE APPROXIMATE CONTINUITY OF VECTOR FUNCTIONS

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**Abstract.** We prove an equivalent condition to the approximate continuity with respect to some differentiation bases of vector functions. This condition is an counterpart of Lipiński condition from [12] for real functions  $f : \mathbb{R} \rightarrow \mathbb{R}$ .

**Key Words and Phrases:** Differentiation basis, approximate continuity, Bochner integrability, derivatives, fixed point.

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