

## EXISTENCE OF POSITIVE MONOTONIC SOLUTIONS OF FUNCTIONAL HYBRID FRACTIONAL INTEGRAL EQUATIONS OF QUADRATIC TYPE

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**Abstract.** We present an existence result for positive monotonic solutions for a certain nonlinear functional hybrid integral equation of quadratic type via fixed point theoretic technique of Dhage [10, 11] in Banach algebras. Our result generalizes the existence result proved in Darwish and Ntouyas [7] and thereby several results as special cases with a different method.

**Key Words and Phrases:** Fractional integral equation, initial value problems, hybrid fixed point theorem, positive solution.

**2010 Mathematics Subject Classification:** 47H10, 34A12, 34A38.

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*Received: April 8, 2013; Accepted: January 23, 2014.*