

EXISTENCE OF MILD SOLUTIONS FOR FRACTIONAL EVOLUTION EQUATIONS

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Abstract. In this article, we establish sufficient conditions for the existence of mild solutions for fractional evolution differential equations by using a new fixed point theorem. The results obtained here improve and generalize many known results. An example is also given to illustrate our results.

Key Words and Phrases: Existence, fractional evolution equations, mild solution, measure of noncompactness, fixed points.

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