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COUPLING EXTRAGRADIENT METHODS WITH CQ METHODS FOR EQUILIBRIUM POINTS, PSEUDOMONOTONE VARIATIONAL INEQUALITIES AND FIXED POINTS

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Abstract. In this paper, we suggest a hybrid method for finding a common element of the set of solution of an equilibrium problem, the set of solution of a pseudomonotone variational inequality problem and the set of common fixed points of an infinite family of nonexpansive mappings. The constructed iterative method combines two well-known methods: extragradient method and *CQ* method. We derive a necessary and sufficient condition for the strong convergence of the sequences generated by the proposed method.

Key Words and Phrases: Equilibrium problem, pseudomonotone variational inequality, fixed point, pseudomonotone mapping, nonexpansive mapping, extragradient method, *CQ* method.

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