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WEAKLY PICARD OPERATORS METHOD FOR MODIFIED FRACTIONAL ITERATIVE FUNCTIONAL DIFFERENTIAL EQUATIONS

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Abstract. Boundary value problems for modified fractional iterative functional differential equations is offered. Using weakly Picard operators method, some new existence and uniqueness theorems and data dependence results are presented. Further, examples are given to illustrate our results.
Key Words and Phrases: Weakly Picard operators, fractional iterative functional differential equations, boundary value problems.

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