A NEW METHOD IN THE STUDY OF IMPULSIVE FRACTIONAL DIFFERENTIAL EQUATIONS IN BANACH SPACES

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Abstract. In this paper, a new method named by Picard and weakly Picard operators methods is applied to revisit impulsive fractional differential equations in Banach spaces. By using this powerful tool, some interesting existence and uniqueness theorems and data dependence results are established in a suitable piecewise continuous functions space with piecewise Bielecki norm.

Key Words and Phrases: Impulsive fractional differential equations, solvability, data dependence, Picard and weakly Picard operators.

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