

ON NONLOCAL INTEGRAL BOUNDARY VALUE PROBLEMS FOR IMPULSIVE NONLINEAR DIFFERENTIAL EQUATIONS OF FRACTIONAL ORDER

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Abstract. In this paper, we investigate the existence and uniqueness of solutions for impulsive nonlinear differential equations of fractional order with nonlocal integral boundary condition. Our results are based on some suitable fixed point theorems. An illustrative example is presented.

Key Words and Phrases: Nonlocal integral boundary condition, impulse, nonlinear differential equations, fractional derivative, fixed point theorem.

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