

## KY FAN'S INEQUALITIES FOR VECTOR-VALUED MULTIFUNCTIONS IN TOPOLOGICAL ORDERED SPACES

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**Abstract.** The aim of this paper is using the cone semicontinuity and cone quasiconvexity for multi-valued mappings to present four variants of Ky Fan's type inequality for vector-valued multifunctions in topological ordered spaces.

**Key Words and Phrases:** fixed point theorem, multivalued mapping, Ky Fan minimax inequality, topological semilattices,  $C_{\Delta}$ -quasiconvex (quasiconcave),  $C$ -upper (lower) semicontinuous.

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