

BOUNDEDNESS AND GLOBAL ATTRACTIVITY FOR A CLASS OF FUNCTIONAL DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we are concerned with the boundedness and global attractivity of some vector functional differential equations. Using fixed point technique instead of the usual Lyapunov direct method, some quite general criteria for boundedness and attractivity are established. We then apply these results to some special delay equations, for which some existing results are included.

Key Words and Phrases: Nonlinear functional differential equation, contraction mapping principle, boundedness, global attractivity.

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