FIXED POINT THEOREMS AND APPLICATIONS IN THEORY OF GAMES

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Abstract. We introduce the notions of weakly *-concave and weakly naturally quasi-concave correspondence and prove fixed point theorems and continuous selection theorems for these kind of correspondences. As applications in the game theory, by using a tehnique based on a continuous selection, we establish new existence results for the equilibrium of the abstract economies. The constraint correspondences are weakly naturally quasi-concave. We show that the equilibrium exists without continuity assumptions.

Key Words and Phrases: weakly naturally quasi-concave correspondence, fixed point theorem, continuous selection, abstract economy, equilibrium.

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