SOLVABILITY OF A FUNCTIONAL EQUATION ARISING IN DYNAMIC PROGRAMMING

ZEQING LIU*, LING GUAN*, SHIN MIN KANG** AND YOUNG CHEL KWUN***

*Department of Mathematics, Liaoning Normal University
Dalian, Liaoning 116029, People's Republic of China
E-mails: zeqingliu@163.com; lingguan@sina.cn

**Department of Mathematics and RINS
Gyeongsang National University, Jinju 660-701, Korea
E-mail: smkang@gnu.ac.kr

***Department of Mathematics, Dong-A University
Pusan 614-714, Korea
E-mail: yckwun@dau.ac.kr

Abstract. This paper deals with solvability of the following functional equation arising in dynamic programming of multistage decision processes

$$f(x) = \underset{y \in D}{\mathrm{opt}} \left\{ u(x,y)(p(x,y) + f(a(x,y))) + v(x,y)\,\mathrm{opt}\{q(x,y), f(b(x,y))\} \right\}, \quad \forall x \in S.$$

Using the Banach fixed point theorem and new iterative techniques, we obtain the existence and uniqueness of solutions for the above equation in the complete metric space $BB(S)$ and the Banach spaces $BC(S)$ and $B(S)$, construct some iterative methods, prove their convergence and provide several error estimates between these iterative sequences generated by the iterative methods and the corresponding solutions, respectively. Four nontrivial examples illustrating applications of the results presented in this paper are provided.
Key Words and Phrases: Dynamic programming, functional equation, Banach fixed point theorem, nonexpansive mapping, iterative methods, error estimates.
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** Corresponding author.

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