

POSITIVE SOLUTIONS FOR A COUPLED SYSTEM OF MIXED HIGHER-ORDER NONLINEAR SINGULAR FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we investigate the existence of positive solutions for a coupled system of mixed higher-order nonlinear singular fractional differential equations with integral boundary conditions. By applying the fixed point theorem on cones and some new general type conditions, some results on the existence of at least one or two positive solutions are obtained.

Key Words and Phrases: Coupled system, fractional differential equations, positive solution, fixed-point theorem.

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