Fixed Point Theory, 15(2014), No. 1, 117-134
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

COINCIDENCES AND FIXED POINTS OF NEW MEIR-KEELER TYPE CONTRACTIONS AND APPLICATIONS

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Abstract. The Meir-Keeler contraction, an important generalization of the classical Banach contraction has received enormous attention during the last four decades. In this paper, we present a review of Meir-Keeler type fixed point theorems and obtain some results using general Meir-Keeler type conditions for a sequence of maps in a metric space. Further, a recent result of Meir-Keeler type common fixed point theorem due to M. Kikkawa and T. Suzuki is generalized under tight minimal conditions. Applications regarding the existence of common solutions of certain functional equations are also discussed.
Key Words and Phrases: Coincidence point, fixed point, Meir-Keeler contraction, functional equation, dynamic programming.
2010 Mathematics Subject Classification: 47H10, 54H25, 49L20.

Acknowledgement. The authors thank Prof. Lj. B. Ciric for his valuable suggestions in this paper. They also thank Dr. Rajendra Pant for his assistance in typing the paper. The second author (SLS) acknowledges the support by the UGC, New Delhi under Emeritus Fellowship.

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Received: April 10, 2012; Accepted: June 21, 2012.