

## A FIXED POINT THEOREM FOR CORRESPONDENCES ON CONE METRIC SPACES

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**Abstract.** In this paper, we prove that if  $f$  is a contractive closed-valued correspondence on a cone metric space  $(X, d)$  and there is a contractive orbit  $\{x_n\}$  for  $f$  at  $x_0 \in X$  such that both  $\{x_{n_i}\}$  and  $\{x_{n_i+1}\}$  converge for some subsequence  $\{x_{n_i}\}$  of  $\{x_n\}$ , then  $f$  has a fixed point, which generalizes a fixed point theorem for contractive closed-valued correspondences from metric spaces to cone metric spaces.

**Key Words and Phrases:** Cone metric space, fixed point, contractive correspondence, closed-valued correspondence, contractive orbit.

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