*Fixed Point Theory*, 15(2014), No. 1, 23-32
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

# EXISTENCE OF SOLUTIONS FOR SECOND ORDER IMPULSIVE CONTROL PROBLEMS WITH BOUNDARY CONDITIONS

ABDELKADER BOUCHERIF*, ALI S. AL-QAHTANI** AND BILAL CHANANE***

King Fahd University of Petroleum and Minerals,
Department of Mathematics and Statistics,
P.O. Box 5046, Dhahran 31261, Saudi Arabia
E-mails: * aboucher@kfupm.edu.sa, **alitalhan@hotmail.com, ***chanane@kfupm.edu.sa

**Abstract.** Control of impulsive differential equations appear naturally in physical phenomena. Most often these phenomena take place during a finite time interval. This leads to the study of boundary value problems for control of impulsive differential equations. In this paper we address the problem of existence of solutions of control of impulsive differential equations of second order subjected to two-point boundary conditions. Our approach is based on the Granas topological transversality theorem and the Schauder fixed point theorem. The uniqueness of solutions is also discussed.
**Key Words and Phrases**: second order impulsive control problem, boundary value problems, Granas topological transversality theorem, Schauder fixed point theorem.

**2010 Mathematics Subject Classification**: 37B37, 34B15, 34H05, 47N20, 93C15, 47H10.

**Acknowledgement.** The authors would like to thank an anonymous referee for a careful reading of the manuscript. Also, they are grateful to King Fahd University of Petroleum and Minerals for its constant support.

## References

[1] B. Ahmad, *Existence of solutions for second-order nonlinear impulsive boundary-value problems*, Elect. J. Diff. Eq., 2009, 1–7.
[2] D.D. Bainov, P.S. Simeonov, *Impulsive Differential Dquations: Periodic Solutions and Applications*, Longman Scientific and Technical, Essex, England, 1993.
[3] L.H. Erbe, X. Liu, *Existence results for boundary value problems of second order impulsive differential equations*, J. Math. Anal. Appl., **149**(1990), 56-69.
[4] A. Granas, J. Dugundji, *Fixed Point Theory, Springer*, New York, 2003.
[5] E.R. Kaufmann, N. Kosmatov, Y.N. Raffoul, *A second-order boundary value problem with impulsive effects on an unbounded domain*, Nonlinear Anal., **69**(2008), 2924–2929.
[6] A. Lakmeche, A. Boucherif, *Boundary value problems for impulsive second order differential equations*, Dynam. Cont. Discr. Impul. Syst. Series A, Math. Anal., **9**(2002), 313-319.
[7] V. Lakshmikantham, D.D. Bainov, P.S. Simeonov, *Theory of Impulsive Differential Equations*, World Scientific, Singapore, 1989.
[8] Y. Lee, X. Liu, *Study of singular boundary value problems for second order impulsive differential equations*, J. Math. Anal. Appl., **331**(2007), 159-176.
[9] V.D. Milman, A.D. Myshkis, *On the stability of motion in the presence of impulses* (Russian), Sibirsk. Mat. Z., **1**(1960), 233-237.

[10] J.J. Nieto, *Basic theory for nonresonance impulsive periodic problems of first order*, J. Math. Anal. Appl., **205**(1997), 423-433.
[11] I. Rachůnková, J. Tomeček, *Singular Dirichlet problem for ordinary differential equations with impulses*, Nonlinear Anal., **65**(2006), 210-229.
[12] A.M. Samoilenko, N.A. Perestyuk, *Impulsive Differential Equations*, World Scientific, Singapore, 1995.
[13] J. Tomeček, *Nonlinear boundary value problem for nonlinear second order differential equations with impulses*, Electronic J. Qual. Th. Diff. Eq., **10**(2005), 1-22.
[14] T. Yang, *Impulsive Control Theory*, Springer, Berlin, 2001.

*Received: March 28, 2012; Accepted: January 17, 2013.*