

MULTIVALUED FIXED POINT THEOREMS WITHOUT STRONG COMPACTNESS VIA A GENERALIZATION OF MIDPOINT CONVEXITY

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Abstract. We investigate a generalization of midpoint convexity for multivalued maps, and derive fixed points theorems under this more general assumption without requiring any strong compactness condition. As an application we prove the existence of social equilibria for games with n players.

Key Words and Phrases: Fixed points; midpoint convex multimaps; midpoint linear maps and multimaps; n -players games.

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