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THE EXISTENCE OF MULTIPLE FIXED POINTS FOR THE SUM OF TWO OPERATORS AND APPLICATIONS

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Abstract. In this paper, by means of τ - φ -concave (convex) operators, the existence of two positive fixed points for some nonlinear operators is considered. In particular, the fixed point theorems of the sum of a φ_1 -concave operator and a φ_2 -convex operator are obtained, our tools are based on the properties of cones and the fixed point theorem of cone expansion and compression. Our abstract results are applied to superlinear second-order multi-point boundary value problems.

Key Words and Phrases: Cone, τ - φ -concave operator, τ - φ -convex operator, superlinear, multipoint boundary value problem.

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