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A NEW MODEL OF PRICE FLUCTUATION FOR A SINGLE COMMODITY MARKET

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Abstract. In this paper we study a new model for the dynamics of price fluctuation in a single commodity market of the form (7)+(8), which continues the results of the [1], [2], [3]. The model is formulated as a following Wheeler-Feynman problem:

(1) $$x'(t) = f(t, x(t), x(t-h), x(t+h)), \quad t \in \mathbb{R}$$

(2) $$x(t) = \varphi(t), \quad t \in [t_0 - h, t_0 + h],$$

where $t_0 \in \mathbb{R}$, $h > 0$, $f \in C(\mathbb{R}^4)$, $\varphi \in C^1[t_0 - h, t_0 + h]$.

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1. Introduction

J. Belair and M.C. Mackey [1] considered a model in order to study the dynamics of price, production and consumption for a particular commodity governed by the equation

(1) $$\frac{1}{P}\frac{dP}{dt} = f(D(P_D), S(P_S)).$$

Here $D(\cdot)$ is the demand function, $S(\cdot)$ is the supply function, P_D is the price (demand) established by the dynamics of consumption and P_S is the price (supply) established on the dynamics of supply.

Belair and Mackey compared this model with other models that have been studied in economic literature and arrived at the conclusion that the following J.B.S. Haldane's model (1933):

(2) $$\frac{dp}{dt} = -Ap - B\int_0^\infty g(x)p(t-x)dx,$$

where p is the deviation of commodity price of equilibrium value, is a special case of their model.

A.M. Farahani and E.A. Grove [2] obtained sufficient and also necessary and sufficient conditions for all positive solutions to oscillate about the unique positive steady state solution of equation

(3) $$\frac{P'(t)}{P(t)} = \frac{a}{b + P^n(t)} - \frac{cP^m(t-\tau)}{d + P^m(t-\tau)},$$

$a, b, c, d, \tau, m \in (0, \infty)$ and $n \in [1, \infty)$.

A.S. Mureşan [4] studied a special case of fluctuation model for the price with retard of the form

$$p'(t) = p(t)\left(\frac{a}{b + p^q(t)} - \frac{cp^r(g(t))}{d + p^r(g(t))}\right) \tag{4}$$

and proved that there exists a positive, bounded, unique solution.

I.A. Rus and C. Iancu [5] studied a more general model of the form

$$x'(t) = F(x(t), x(t-\tau))x(t), \quad t \in \mathbb{R} \tag{5}$$

$$x(t) = \varphi(t), \quad t \in [-\tau, 0]. \tag{6}$$

They proved the existence and uniqueness for the solution x^* of the problem (5)+(6) and established some relations between equilibrium solution and coincidence points.

2. Our model

In this paper we consider the following model

$$x'(t) = x(t)[D(x(t)) - S(x(t-h), x(t+h))], \quad t \in \mathbb{R} \tag{7}$$

$$x(t) = \varphi(t), \quad t \in [t_0 - h, t_0 + h], \tag{8}$$

t_0 - the initial moment, $h > 0$, $D \in C(\mathbb{R}_+, \mathbb{R}_+)$, $S \in C(\mathbb{R}_+ \times \mathbb{R}_+, \mathbb{R}_+)$.

Here

$$D(x(t)) = \frac{a}{b + x^n(t)},$$
$$S(x(t-h), x(t+h)) = S_1(x(t-h)) - S_2(x(t+h))$$
$$S_1(x(t-h)) = \frac{cx^m(t-h)}{d + x^m(t-h)},$$
$$S_2(x(t+h)) = \frac{kx^p(t+h)}{r + x^p(t+h)},$$

$t \in \mathbb{R}$, $h > 0$, $a, b, c, d, k, r > 0$.

In the relationship of S_1 we take the relative speed of variation for supply function before the reference moment t, with the length $h > 0$. In the relationship of S_2 we take the relative speed of variation for supply function that can be after the same lenght $h > 0$ after the reference moment t.

In this model $E(t, x(t)) = x(t)D(x(t))$ represents the elasticity function of demand, in respect to price, when the demand has a linear dependence in respect to price. Consequently $D(x(t))$ is the relative speed of variation for the demand function and has the economic semnification as the elasticity of monetary unity for price $(E(t, x(t))/x(t))$.

Our model (7)+(8) is more general as that studied in [4], in which appear only the retard of argument, by considering a retard and an advanced argument. Thus one obtains the model

$$x'(t) = f(t, x(t), x(t-h), x(t+h)), \quad t \in \mathbb{R} \tag{9}$$

$$x(t) = \varphi(t), \quad t \in [t_0 - h, t_0 + h], \tag{10}$$

$t_0 \in \mathbb{R}$, $h > 0$, $f \in C(\mathbb{R}^4)$, $\varphi \in C^1[t_0 - h, t_0 + h]$ which have been studied in [6].

3. The main results

For the model (7)+(8) we have

Theorem 3.1. *If the conditions*

$$k - c < 0, \quad b(k - c) + a < 0, \quad a - cb < 0 \tag{11}$$

are satisfied then there exists an equilibrium positive solution of the equation (7).

Moreover, there exists a number $u_0 \geq 0$ such that, on the interval $[u_0, +\infty)$, the equilibrium solution of the equation (7) is unique.

Proof. Economic reasons lead us to the following inequality

$$S(x(t - h), x(t + h)) > 0 \text{ for all } t \in \mathbb{R},\ h > 0.$$

Thus it results that $c > k$.

Then, let u be the equilibrium solution of the equation (7), which is independent of t. We obtain for the equilibrium solution the following equation

$$\frac{a}{b + u^n} - \frac{cu^m}{d + u^m} + \frac{ku^p}{r + u^p} = 0$$

or

$$(k - c)u^{m+n+p} + (kb - cb + a)u^{m+p} + kdu^{n+p} - cru^{m+n} + \tag{12}$$
$$+ d(a + kb)u^p + r(a - cb)u^m + adr = 0.$$

The conditions (11) assure that this equation has at least one positive solution u^*. So this u^* is an equilibrium solution of the equation (7).

We denote by $h(u)$ the left side of the relationship (12) and let u_0 be the following number

$$u_0 = \sup\{u |\ u \in [0, +\infty),\ h(u) > 0\}.$$

We have $h(u_0) > 0$ and $\lim_{u \to \infty} h(u) = -\infty$, thus there exists a unique solution u^* of the equation (7) on the interval $[u_0, +\infty)$ which is an equilibrium positive solution.

Remark. When $m = n = p$, $k - c < 0$ and $rk < a + cd$, the uniqueness of the equilibrium solution of the equation (7) is assured.

Theorem 3.2. *If $x \in C^1(\mathbb{R}, \mathbb{R}_+)$ is a solution of the equation (7) then $x \in C^\infty(\mathbb{R}, \mathbb{R}_+)$.*

Proof. The function f given by

$$f(t, u, v, w) = u[D(u) - S(v, w)],$$

is in $C^\infty(\mathbb{R}^4)$. Here

$$f(t, x(t), x(t - h), x(t + h)) = x(t)[D(x(t)) - S(x(t - h), x(t + h))]$$

So, by induction and by applying the Theorem 3.1 from [6] it follows that $x \in C^n(\mathbb{R}, \mathbb{R}_+)$ for all $n \in \mathbb{N}$.

Theorem 3.3. *We suppose that the following conditions*

$$c \neq 1, \quad k \neq 1, \quad \varphi \in C^\infty[t_0 - h, t_0 + h],$$

are satisfied. Then the problem (7)+(8) has a unique solution if and only if φ satisfies the relations

$$\varphi^{(n+1)}(t_0) = [f(t, \varphi(t), \varphi(t-h), \varphi(t+h))]^{(n)}_{t=t_0}, \ (\forall) \ n \in \mathbb{N}.$$

Proof. We can use the Theorem 3.2 from [6] because the function $f \in C^{\infty}(\mathbb{R}^4)$ and there are satisfied the hypothesis (H_1) and (H_2) of this theorem.

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