DIDACTICA MATHEMATICA, Vol. 34(2016), pp. 41–49

NATURAL CONVECTION FLOW IN A VERTICAL CHANNEL IN THE PRESENCE OF RADIATION AND VISCOUS DISSIPATION

Septimiu Nicolae Murar

Abstract. This paper investigates the effects of radiation and viscous dissipation on the steady free convection flow in a vertical channel for laminar and fully developed flow regime. The Rosseland approximation is considered in the modeling of the convection-radiation heat transfer and the temperature of the walls are assumed constant. The governing equations are expressed in non-dimensional form and are solved numerically using the central finite difference method and the Matlab solver bvp4c.

MSC 2000. 76R10, 76D05, 34B15.

Key words. Natural convection, Thermal radiation, Viscous dissipation, Vertical channel

1. INTRODUCTION

Heat transfer in free and mixed convection in vertical channels has been the subject of many detailed, mostly numerical, studies for different flow configuration. The interest in this subject is due to its applications, for example, in the design of cooling systems for electronic devices, chemical processing equipment, microelectronic cooling and in the field of solar energy collection. Some of the published papers, such as by Aung(see [1]), Aung and Worku(see [2] and [3]), Barlleta(see [4]) and Boulama and Galanis(see [5]), deal with the evaluation of the temperature and velocity profiles for the vertical parallel-flow fully developed regime. In the above quoted papers the thermal radiation effect within the fluid is neglected.

Heat transfer by simultaneous radiation and convection is very important in the processes involving high temperatures, in the context of space technology and in numerous technological problems, including combustion, furnace design, the design of high-temperature gas-cooled nuclear reactors, nuclear-reactor safety, solar collectors, and many others. The inclusion of convection-radiation effects in the energy equation leads to a highly nonlinear partial or ordinary differential equations. The analysis of thermal radiation is complicated due to the behavior of the radiative properties of materials. Properties relevant to conduction and convection, including, thermal conductivity, kinematic viscosity, density are fairly easily measured and generally well behaved. For more information about the radiative heat transfer, its practical applications and its interactions with conduction and convection the reader can consult the book [6].

In the following lines we investigate the effects of thermal radiation and viscous dissipation on the steady fully developed free convection flow in a vertical channel whose walls are subjected to uniform but different temperatures. We will use the Rosseland approximation model which leads to an ordinary differential equations for an optically dense viscous incompressible fluid that flows through the channel. The ordinary differential equations are solved analytically for a particular case when we consider the dimensionless numbers Rd and Ec to be zero and numerically while Rd varies. Effects of parameters such as the radiation parameter, Rd, the temperature parameter, θ_R, the convection parameter, Ra, on velocity and temperature profiles, are shown graphically.

2. MATHEMATICAL MODEL

Consider a viscous and incompressible fluid, which steadily flows between two infinite vertical and parallel plane walls. The channel width is L. A coordinate system is chosen such that the x-axis is parallel to the gravitational acceleration vector g, but with the opposite direction. The y-axis is orthogonal to the channel walls, and the origin of the axis is such that the positions of the channel walls are in $y = 0$ and $y = L$, respectively see Figure 2.1. The wall at $y = 0$ has the given temperature T_h, and the wall at $y = L$ has the given temperature T_c, where $T_h > T_c$. Since the fluid velocity vector $v(u, v)$ is assumed to be parallel to the x-axis v vanish. The Boussinesq and Rosseland approximation are employed. The fluid flow is due to difference in temperature (buoyancy force) and initial velocity U_0.

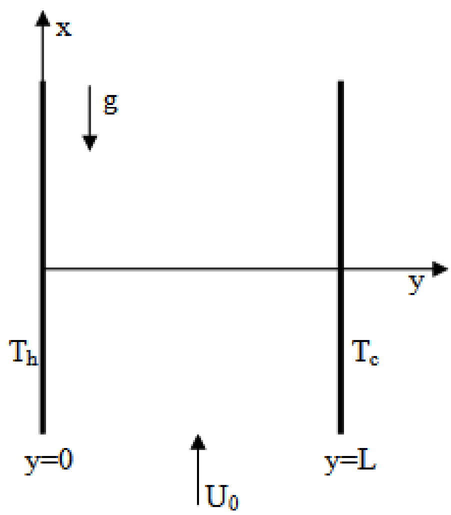

FIG. 2.1 – Geometry of the problem

From the assumption of free convection and fully developed flow the following relations are true (see [7, p. 37-47]):

$$v = 0, \quad \nabla p = 0, \quad \frac{\partial T}{\partial x} = 0, \quad \frac{\partial q_r}{\partial x} = 0 \tag{1}$$

where p is the fluid pressure, T is the temperature of the fluid and q^r is the radiation heat flux. Replacing (1) in the Navier-Stokes equation and in the energy equation we obtain the governing equations for our problem:

$$\mu \frac{\partial^2 u}{\partial y^2} + \rho_0 g \beta (T - T_0) = 0 \tag{2}$$

$$\frac{\partial}{\partial y}\left[\left(\alpha + \frac{1}{\rho C_p}\frac{16\sigma T^3}{3K_{\text{ROSS}}}\right)\frac{\partial T}{\partial y}\right] + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 = 0 \tag{3}$$

where α is the thermal diffusivity coefficient, β is the thermal expansion coefficient, μ is the dynamic viscosity, ρ_0 is the characteristic density of the fluid and $T_0 = \dfrac{T_h + T_c}{2}$.

In (3) we have assumed that q^r under the Rosseland approximation has the following form (see [6, Section 14.2]):

$$q^r = -\frac{4\sigma}{3K_{\text{ROSS}}}\frac{\partial T^4}{\partial y} = -\frac{16\sigma T^3}{3K_{\text{ROSS}}}\frac{\partial T}{\partial y}$$

where σ is the Stefan-Boltzman constant and K_{ROSS} is the mean absorption coefficient. Equations (2) and (3) have to be solved subject to the boundary conditions.

$$u(0) = 0, \quad u(L) = 0, \quad T(0) = T_h, \quad T(L) = T_c \tag{4}$$

Further we will introduce the following non-dimensional variables

$$Y = \frac{y}{L}, \quad U(Y) = \frac{u}{U_0}, \quad \theta(Y) = \frac{T - T_0}{T_h - T_c} \tag{5}$$

where $T_0 = \dfrac{T_h + T_c}{2}$ and we consider $U_0 = \dfrac{\alpha}{L}$. Substituting (5) into equations (2) and (3) we will obtain the following non-dimensional ordinary differential equations:

$$\frac{\partial^2 U}{\partial Y^2} + Ra\,\theta = 0 \tag{6}$$

$$\frac{\partial}{\partial Y}\left[\left(1 + \frac{4}{3}Rd(1 + 2(\theta_R - 1)\theta)^3\right)\frac{\partial \theta}{\partial Y}\right] + EcPr\left(\frac{\partial U}{\partial Y}\right)^2 = 0 \tag{7}$$

The boundary conditions (4) will become in dimensionless form:

$$U(0) = 0, \quad U(1) = 0, \quad \theta(0) = \frac{1}{2}, \quad \theta(1) = -\frac{1}{2} \tag{8}$$

Here Ra is the Rayleigh number, Rd is the radiation parameter, θ_R is the temperature parameter, Ec is the Eckert number and Pr is the Prandtl number defined as:

$$(9)\qquad Ra = \frac{g\beta\Delta T L^3}{\alpha\nu},\ Rd = \frac{4\sigma T_0^3}{k\,K_{\mathrm{ROSS}}},\ \theta_R = \frac{T_h}{T_0},\ Ec = \frac{U_0^2}{c_P\Delta T},\ Pr = \frac{c_P\mu}{\alpha}$$

where k is the thermal conductivity of the fluid

We notice that in the case when the radiation and viscous dissipation effects are absent ($Rd = 0$, $Ec = 0$) our problem has an analytical solution which can be expressed as:

$$(10)\qquad U(Y) = Ra\left(\frac{1}{6}Y^3 - \frac{1}{4}Y^2 + \frac{1}{12}Y\right),\quad \theta(Y) = -Y + \frac{1}{2}$$

The physical quantities of interest in this problem are the Nusselt numbers which are defined as:

$$(11)\qquad Nu = \frac{h_w L}{k}$$

where the convective heat flux coefficient at the walls, h_w are given by:

$$(12)\qquad -k\frac{\partial T}{\partial y}\bigg|_{y=0} + q^r|_{y=0} = h_w[T_h - T_c]$$

Using (5), (11) and (12) we obtain

$$Nu_1 = -\left(1 + \frac{4}{3}Rd\,\theta_R^3\right)\left(\frac{\partial\theta}{\partial Y}\right)\bigg|_{Y=0}$$

Similarly if we consider in (12) $y = L$ we obtain another Nusselt number Nu_2.

$$Nu_2 = \left(1 + \frac{4}{3}Rd(2-\theta_R)^3\right)\left(\frac{\partial\theta}{\partial Y}\right)\bigg|_{Y=L}$$

3. RESULTS AND DISCUSSION

Equations (6) and (7) with the boundary conditions (8) were solved numerically for different values of parameters Ra, Rd, θ_R and Ec ($Ra = 10, 15, 20, 25,$ $250, 500, 750, 1000$, $Rd = 0, 0.1, 1, 5, 10$, $\theta_R = 1.1, 1.5, 2$, $Ec = 0, 0.01$ and $Pr = 0.71$) using two methods, namely, a central finite-difference method and the Matlab solver bvp4c. It was found that in the case of $Ec = 0$ both Nusselt numbers Nu_1 and Nu_2 are equal and our results for Nu_1 is very close to the results obtain by T. Grosan and I. Pop (see [7]) . Therefore we are confident that the present results are accurate. It can be seen in the following table that the value of Nu_1, in the case when $Ec = 0$, increases with the increases of the radiation parameter Rd and the temperature parameter θ_R.

Rd	θ_R	T. Grosan and I. Pop [7]	Present result
1	1.1	2.346	2.3467
	1.5	2.666	2.6667
	2	3.667	3.6667
5	1.1	7.733	7.7334
	1.5	9.318	9.3333
	2	14.334	14.3334
10	1.1	14.465	14.4667
	1.5	17.613	17.6667
	2	27.668	27.6668

In the following table we present the values for both Nusselt numbers Nu_1 and Nu_2 for the same values of parameters Rd and θ_R as in the above table, and for $Ec = 0.01$. It can be seen that the differences between Nu_1 and Nu_2 increase with the increasing of the parameters Rd and θ_R.

Rd	θ_R	Nu_1	Nu_2
1	1.1	2.2807	2.3865
	1.5	2.4997	2.7320
	2	3.3769	3.8244
5	1.1	7.6588	7.7711
	1.5	9.1050	9.4417
	2	13.9778	14.5660
10	1.1	14.3904	14.5041
	1.5	17.4272	17.7847
	2	27.3018	27.9222

Dimensionless temperature profiles are presented in Figure 3.2 and 3.3. We notice that the thickness of the temperature profiles increase with the increasing of the parameters Rd and θ_R. The velocity profiles are presented in Figure 3.4, 3.5, 3.6 and 3.7. The analytical solution given by (10) is also included in Figure 3.4 and the agreement with the numerical solutions is very good. It can be seen that the agreement between the numerical solutions obtained by the central finite difference method and bvp4c is also very good.

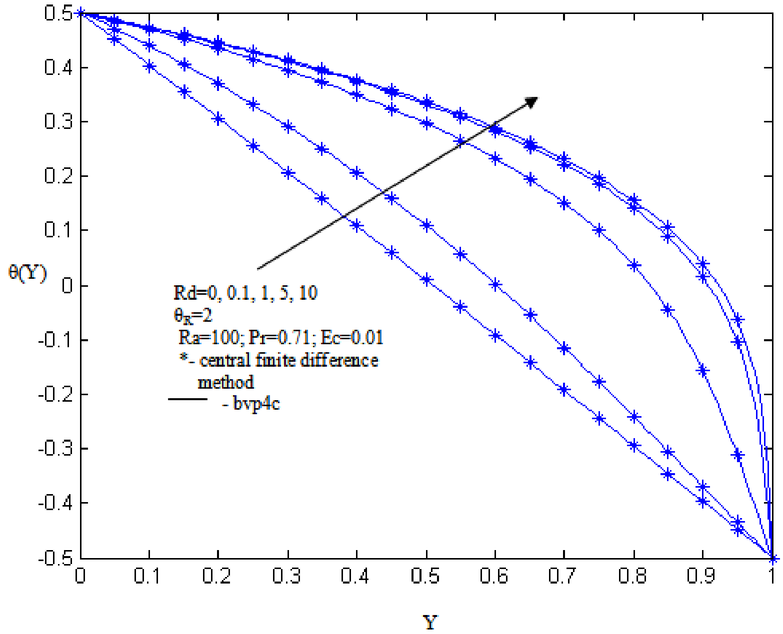

FIG. 3.2 – Dimensionless temperature profiles for different values of parameter *Rd*.

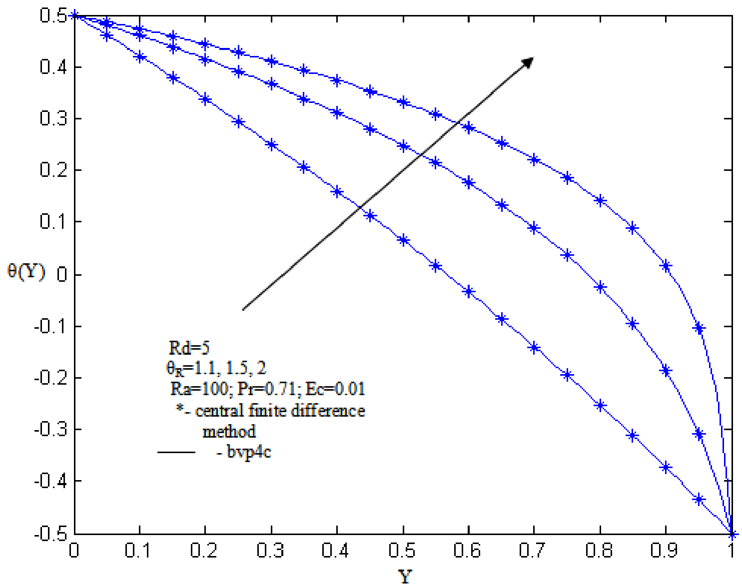

FIG. 3.3 – Dimensionless temperature profiles for different values of parameter θ_R.

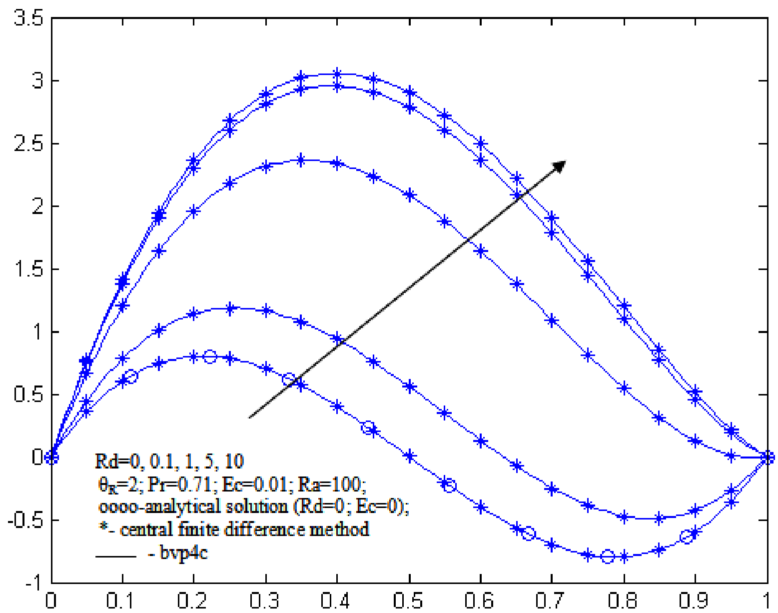

FIG. 3.4 – Dimensionless velocity profiles for different values of parameter Rd.

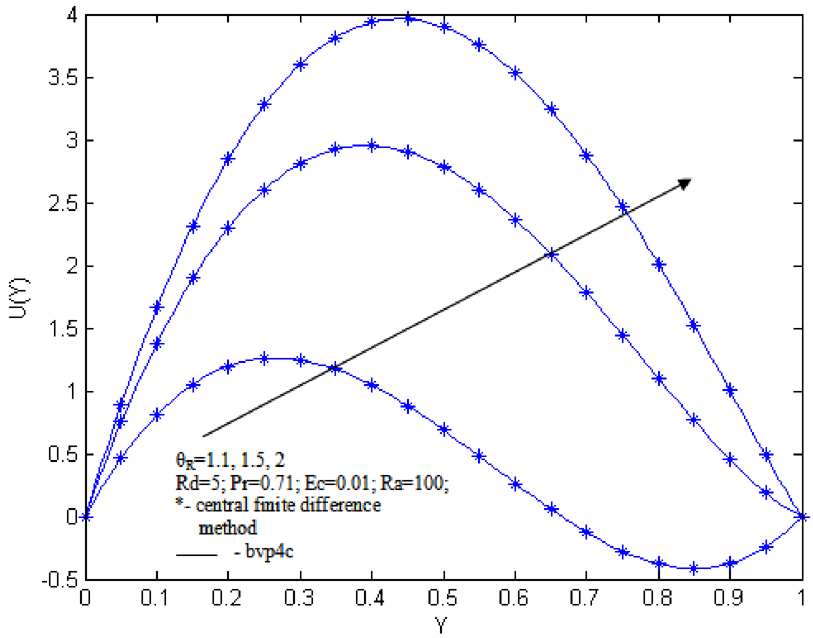

FIG. 3.5 – Dimensionless velocity profiles for different values of parameter θ_R.

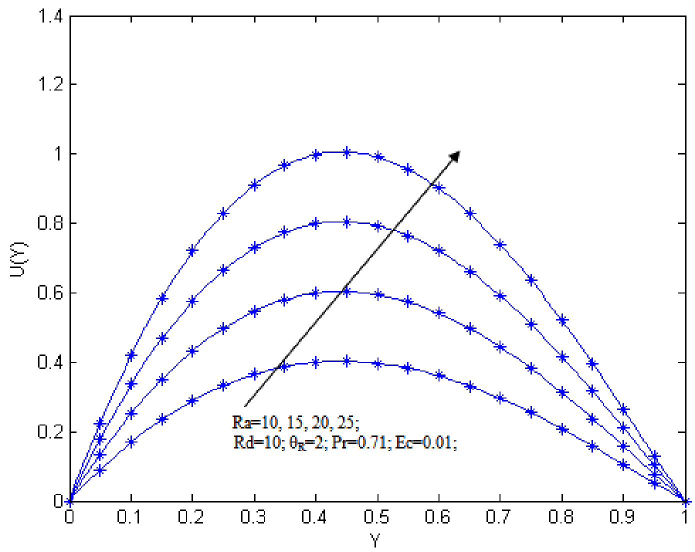

FIG. 3.6 – Dimensionless velocity profiles for small values of parameter Ra.

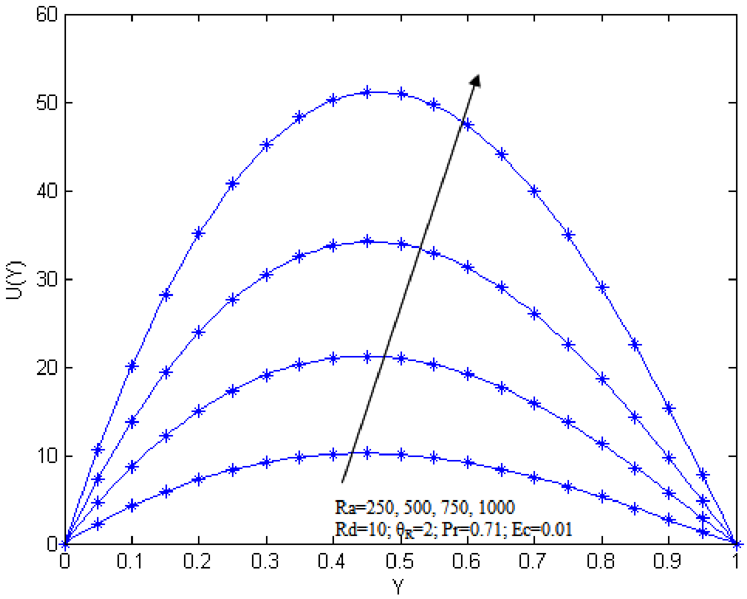

FIG. 3.7 – Dimensionless velocity profiles for large values of parameter *Ra*.

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Faculty of Mathematics and Computer Science
Babeş-Bolyai University Cluj-Napoca, Romania
e-mail: `septimiumurar@yahoo.com`

Received: 22 Mai 2016